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Evaluation of Methods for Determination of Dispersion Coefficient for Utu Etim-Ekpo River, Nigeria

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Authors' contributions

This work was carried out in collaboration between all authors. Author ILN served as research supervisor, designed the study and guided the analysis procedures. Author HAA carried out the literature survey, wrote the protocol as well as the first draft of this manuscript. Authors HAA and OB managed the analysis of the study. Author OB performed the statistical analysis. All authors read and approved the final manuscript.

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ABSTRACT

This study was conducted to evaluate methods for the determination of the longitudinal dispersion coefficient of Utu Etim-Ekpo River. Rihodamine-B was used in the tracer dye study. The variable distance – variable time method of sample collection was adopted. The samples collected were immediately transferred to the laboratory for analysis. Three approaches were adopted for the computation of dispersion coefficient with respect to this study, namely: Leverspiel & Smith method; Agunwamba's method; and Ojiako's method, respectively. The first two methods are analytical while the third is numerical. The dispersion coefficients obtained were 16.24, 19.679 and 14.68 m²/s with respect to Leverspiel & Smith method, Agunwamba's method and Ojiako's method, respectively. Agunwamba gave a higher value than the other methods due to the fact that the model is best suited for when tracer experiments are done using variable distance – variable time method which was not the case in Utu Etim-Ekpo tracer dye study.

Keywords: Longitudinal dispersion coefficient; Utu Etim-Ekpo river; Rihodamine-B; variable-distance - time.

1. INTRODUCTION

River pollution and transport of pollutant has been a matter of concern and much studies over the years. Pollutant that finds its way into the river could be of different types. It may be accidental spill of toxic substances or it could be combined sewer intentionally discharged into the river. Stream of pollutants, when discharged into a river is subjected to different stages of mixing as the current of flowing water transports these downstream. In the early stages of the transport process after the pollutant has been discharged into the river, advection plays a significant role in the transportation of pollutants. In the later stages, when the cross-sectional mixing must have been completed, the process of longitudinal dispersion becomes important. A number of hvdrodvnamic parameters influence longitudinal dispersion behaviour in a river [1]. In a river (natural channel), the predominant process of mixing of contaminant is the longitudinal dispersion. The importance of dispersion coefficient in the formulation of model to predict concentration of contaminant in a river can never be over-emphasized. The adequate estimation of dispersion coefficient in any given river of interest is very essential for the water Engineers, Environmental Health Officers as well as Environmental Managers. It has been noted that the estimation of dispersion coefficient is a very tasking endeavor as various parameters lead to the mixing process considering that river reaches may vary in conditions; one formula is therefore not suitable for the estimation of the accurate dispersion coefficient for the whole stretch of the river. However, this is seen as a common approach in the field of hydraulic engineering [2].

Elder [3] published on the longitudinal dispersion coefficient studies. His work was based on laboratory measurements and Taylor's method on oven channel. He assumed a logarithmic vertical-velocity distribution to derive an empirical equation for dispersion coefficient estimation. Taylor [4] presented the equations to predict the longitudinal dispersion coefficient in a steady laminar and turbulent pile flow respectively. He showed that the cross section average concentration distribution can be described in Fick's second law of diffusion. Fischer [5] expanding on Taylor's assumptions for the mass conservation equation for turbulent flow assumed

that transverse or lateral variations are more important. In comparison to the vertical variation in the velocity profile developed a new equation for dispersion coefficient.

Agunwamba [6] developed an analytical equation that can be used for computing the coefficient of dispersion. The equation developed by Agunwamba took care of the need for the concentration of the tracer dye to insignificance at the outlet thereby eliminating the cost in performing the experiment, the time needed for the experiment and the physical effort required to perform the experiment.

2. MATERIALS AND METHODS

2.1 Study Area

Utu Etim - Ekpo in Etim - Ekpo Local Government Area of Akwa Ibom State Nigeria lies within the latitude 4° 0" 34.2" and longitude 7º 37" 11.64" and 8º 48" East of the Greenwich Meridian. Fig. 1 presents the map of Utu Etim -Ekpo. Utu Etim-Ekpo has the tropical humid climate characterized by distinct wet and dry seasons. The wet season extends from March to October while November to February defines the dry season. Utu Etim-Ekpo has a temperature that is nearly constant through-out the year. The temperature ranges from 27°C in July to 29°C in the month of March. The measured river temperature was 28°C with respect to this study. The relative humidity of Utu Etim - Ekpo is high in July to October. The relative humidity could be as high as 96% during peak rainy season and low as 65% in the dry season.

2.2 Method of Data Collection

2.2.1 Hydraulic parameters

The relevant data for Utu Etim-Ekpo river dispersion study include the following: i) Depth of flow; ii) Geometric cross sections; and iii) Velocity of flow. Also, a drone flown in the study area aided in capturing aerial photographs of the river and its environs (see Fig. 2). The sampling was taken at 10 different stations at a distance of 20 metres regular intervals over a stretch of 200 metre length of the river section. The division was carried out using a linear tape and metal poles with reflection ribbon tied on them for easy identifications. The co-ordinate of each of the sampling stations was recorded using hand held

Global positioning system (GPS). The depth of the river flow was determined in the field using manual Echo sounder machine. The machine was fixed to a paddle boat while shooting was done at each of the predetermined sampling stations for the recording of the river depth. The flow velocity of each of the sampling stations was obtained using a floater and stop watch. The experiment at each station was repeated several times and the average flow velocity for each station was determined.

2.2.2 Tracer dye experiment

The experiment was carried out using Rhodamine-B tracer dye. Ten (10) grams of Rhodamine dye was added to 1 litre of water to obtain an initial concentration of 10 mg/litre before it was instantaneously injected into the centre line of the river. The initial concentration of the Rhodamine/water mixture was obtained before the actual injection was done. The injection was done while sitting on the paddle

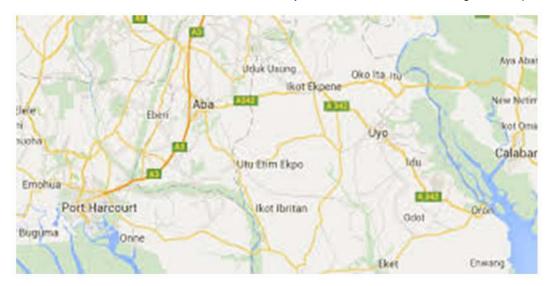


Fig. 1. Google map of Utu Etim-Ekpo Source: Google map [7]



Fig. 2. Aerial photograph of Utu Etim-Ekpo River using Drone (Location at latitude 5,0,24. Longitude 7, 36, 59)

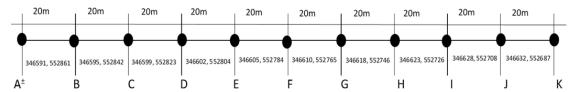


Fig. 3. Sampling station coordinate and hydraulic parameters [‡]A = Point of injection of tracer (Rhodamine-B) at centerline of the river

boat. In view of the high velocity of river flow, 5 different boats with 2 Assistants in each boats were anchored at the 1st five stations before the commencement of the sampling to ensure the accuracy of the sample collection. Another boat aided in the instantaneous injection of the sample and recording of necessary information.

The sample collection commenced following the adequate dilution of the sample by the river. The samples were collected using sterilized plastic containers with proper identification mark and syringe to avoid the disturbance of natural river flow. The samples were collected at a regular time intervals of 3 seconds. The timing was done by one of the field attendants using stop watch. Fig. 3 presents a schematic of the sampling stations. The concentration of samples was obtained by using an instrument called a JENWAY 6305 UV Spectrophotometer. The samples collected were immediately transferred to the laboratory for analysis. The tracer concentration in each of the samples was determined.

2.3 Data Analysis

The longitudinal dispersion coefficient, *D* in a laminar flow was proposed by Taylor [4]. He showed that the cross sectional average concentration distribution can be described by Fick's second law of diffusion with some modification to account for dead zone and skewed nature of concentration-time measurement, a forcing term was added [8] viz:

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} - U \frac{\partial c}{\partial x} + kc \tag{1}$$

Where: c = tracer response concentration (mg/l); D = dispersion coefficient (m²/s); U = mean velocity of flow; t = time (sec); x = distance in the direction of flow; and k = forcing term (sec⁻¹).

Three approaches were adopted for the computation of dispersion coefficient, namely:

i) Leverspiel & Smith method; ii) Agunwamba's method; and Ojiako's method, respectively. The first two methods are analytical while the third is numerical. Details of each method are as presented.

2.3.1 Leverspiel and Smith method

The analytical solution to Equation (1) by Leverspiel & Smith [9] is without the forcing term. The basic principle is the relationship between variance and dispersion which is derived analytically using statistical moment approach as proposed by Leverspiel & Smith [9], we have

$$\delta = \frac{1}{8} \left[\sqrt{8\sigma^2 + 1} - 1 \right] \tag{2a}$$

$$\sigma^{2} = \frac{1}{\theta^{2}} \left[\sum_{i=1}^{n} c_{i} t_{i}^{2} / \sum_{i}^{n} c_{i} - \overline{\theta^{2}} \right]$$
 (2b)

$$\overline{\theta} = \sum_{i=1}^{n} c_i t_i^2 / \sum_{i=1}^{n} c_i$$
 (2c)

$$D = U\delta L$$
 (2d)

Where: t_i = time after injection of tracer (sec); c = tracer response concentration; $\overline{\theta}$ = average flow time given by Marecos do Monte & Mara [10].

2.3.2 Agunwamba's method

Given that Equations (2a - c) are for constant time and distance approach, a modified version for variable distance – time data proposed by Agunwamba [6] as:

$$\delta = \frac{1}{29.2} \left[\sqrt{1 + 15\sigma^2} - 1 \right] \tag{3a}$$

and

$$\sigma^{2} = \left[\sum_{i=1}^{n} \left(\frac{\tau}{1-\xi} \right)^{2} c / \sum_{i=1}^{n} c_{i} \right] - \left[\sum_{i=1}^{n} \left(\frac{\tau}{1-\xi} \right)^{2} c / \sum_{i=1}^{n} c_{i} \right]^{2}$$
 (3b)

Where the summation is taken over all the uniformly spaced readings; $\tau=t/\theta$ and $\xi=x/L$; L = channel length; x = distance from the outlet and t = time after tracer injection.

2.3.3 Ojiako's numerical method

Given that the dispersion coefficient, D and forcing parameter, k are to be determined from Equation (1), Ojiako's approach was to take the logarithm of its analytical solution (Equation 4), Substituting $z = \ln c$ and differentiating yields Equation (5):

$$c = \frac{c_0}{2\sqrt{\pi Dt}} \exp\left[\frac{-(x - Ut)^2}{4Dt} - kt\right]$$
 (4)

and

$$\frac{dz}{dt} = \frac{1}{2t} + \frac{1}{4D} \left(\frac{x^2}{t^2} - \overline{U}^2 \right) - k$$
 (5)

Differentiating Equation (5) again gives a second derivative as Equation (6):

$$\frac{\partial^2 z}{\partial t^2} = -\frac{1}{2t^2} - \frac{x^2}{2Dt^3} \tag{6a}$$

or
$$\frac{t^2 \partial^2 z}{\partial t^2} = -\frac{1}{2} - \frac{x^2}{2Dt}$$
 (6b)

Thus, a linear regression is possible on Equation (6b) by taking $y = t^2 \partial^2 t / \partial t^2$ and x = 1/t, the

slope, a=-x/2D. Once the value of D is determined from the result of regression, k is then evaluated from Equation (5) by regressing $\left(\frac{\partial z}{\partial t} + \frac{1}{2t}\right)$ as y against $1/t^2$ as x. The resulting slope, $a = -\left(k + \overline{U}^2/4D\right)$.

3. RESULTS AND DISCUSSION

3.1 Results

The field data obtained from the tracer experiment (Table 1) and the laboratory results on tracer concentration for various sampling points were plotted as tracer response curve (Fig. 4).

3.1.1 Computation of dispersion coefficient

a) By Leverspiel & Smith method

Table 2 presents the excel program output on computation of D by Leverspiel & Smith method, Evaluation of Equations (2a - d) using values from Table 2 yielded the following results:

$$\overline{\theta} = \frac{3673.65}{27.66} = 132.80$$

$$\sigma^2 = \frac{1}{132.80^2} \left[\frac{701065.8}{27.66} - 132.80^2 \right] = 0.437$$

$$\delta = \frac{1}{8} [\sqrt{(8 \times 0.437) + 1} - 1] = 0.140$$
 (Dispersion number), and

D = δ UL = 0.140 x 0.58 x 200 = 16.245 m²/s Coefficient of dispersion.

Table 1. Result from the laboratory test and field data from tracer experiment

Stations	Distance	Time after	Concentration at	Stream	Cross section of stream		
	x (m)	injection (Sec)	each station (mg/L)	velocity	Area (m²)	Width (m)	Depth (m)
1	0	-	10	-	-	-	-
2	20	30	5.94	0.55	42.90	16.50	2.60
3	40	60	3.80	0.57	48.55	19.42	2.50
4	60	90	3.23	0.83	65.78	26.31	2.50
5	80	120	2.54	0.65	64.77	28.16	2.30
6	100	150	2.38	0.53	65.38	23.35	2.80
7	120	180	2.20	0.46	60.96	21.02	2.90
8	140	210	2.03	0.47	52.61	15.03	3.50
9	160	240	1.94	0.55	33.00	10.00	3.30
10	180	270	1.85	0.55	45.08	18.03	2.50
11	200	300	1.76	0.63	52.26	20.10	2.60
Average				0.58	53.13	19.79	2.75

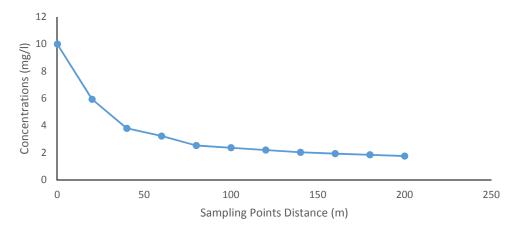


Fig. 4. Tracer response curve for Utu Etim-Ekpo River

Table 2. Excel program output for computation of D using Leverspiel & Smith method

Stations	Distance (m)	Time of release (Sec) t _i	Concentration of tracer dye (mg/l) C _i	$C_i t_i$	C _i t _i ²
1	0	-	10	-	-
2	20	30	5.94	178.125	5343.75
3	40	60	3.80	228	13680
4	60	90	3.23	290.7	26163
5	80	120	2.54	304.38	36525.6
6	100	150	2.38	356.25	53437.5
7	120	180	2.20	396.72	71409.6
8	140	210	2.03	426.93	89655.3
9	160	240	1.94	465.12	111628.8
10	180	270	1.85	500.175	135047.3
11	200	300	1.76	527.25	158175
SUM		·	27.66	3673.65	701065.8

b) By Agunwamba's method

For sake of comparison, Agunwamba's method was applied to the field data, that is, constant distance – time data as against variable distance – time data it is designed for. The summary evaluations of Equations (2c - d) and (3a – b) are as presented in Table 3. To arrive at the value of D, Equations (3a – b) and (2c - d) were evaluated accordingly:-

$$\sigma^2 = \frac{115.86}{27.664} - \left[\frac{57.93}{27.664}\right] = 4.188 - 2.094 = 2.094$$

$$\delta = \frac{1}{27.66} \left[\sqrt{1 + (15 \times 2.094)} - 1 \right] = 0.1694$$
 (Dispersion number); and

 $D = \delta UL = 0.1694 \times 0.58 \times 200 = 19.679 \text{ m}^2/\text{s}$ (Dispersion coefficient).

c) By Ojiako's numerical method

The evaluation of the individual terns of Equations (5) and (6) are summarized in Table 4. The term $\partial z/\partial t$ was evaluated using forward difference approximation, while the second derivative term was evaluated using second order difference approximation, that is, $\partial^2 z/\partial t^2 = \left(z_{i-1} - 2z_i + z_{i+1}/\Delta t^2\right)$

Where i = is the counter for mode number and Δt is the time step and for this study equals 30 seconds. Evaluating Equation (5) with values of applicable terms from Table 3, we have:

$$t^2 \partial^2 z / \partial t^2 = y = -14.985 + 1361.98 \times 1/t$$
 (7)

In which Equation (7), the slope of linear regression is given as:

$$a = \frac{-x^2}{2D} = 1361.98$$
 \Rightarrow $D = \frac{-200^2}{2 \times (1361.98)} = \left| -14.68 \, m^2 / s \right| = 14.68 \, m^2 / s$ (Dispersion coefficient)

Table 3. Excel program output for computation of D using Agunwanba's method

Stations	Distance	Time of release (Sec)	Concentration of tracer dye (mg/l)	C _i t _o	T=t/Θ	E=x/L	1-8	(T/(1-E))*C	(T/(1-E)) ² *C
1	0	-	10	-	-	-	-	-	-
2	20	30	5.94	178.13	0.21	0.9	0.1	12.43	24.87
3	40	60	3.80	228.00	0.42	8.0	0.2	7.96	15.91
4	60	90	3.23	290.70	0.63	0.7	0.3	6.76	13.53
5	80	120	2.54	304.38	0.84	0.6	0.4	5.31	10.62
6	100	150	2.38	356.25	1.05	0.5	0.5	4.97	9.95
7	120	180	2.20	396.72	1.26	0.4	0.6	4.62	9.23
8	140	210	2.03	426.93	1.47	0.3	0.7	4.26	8.51
9	160	240	1.94	465.12	1.68	0.2	8.0	4.06	8.12
10	180	270	1.85	500.18	1.88	0.1	0.9	3.88	7.76
11	200	300	1.76	527.25	2.09	0	1	3.68	7.36
SUM			27.664	3673.65	11.5177			57.93	115.86

Table 4. Excel program output for computation of dispersion coefficient using Numerical method

Stations	Time of release (Sec)	Concentration of tracer dye (mg/l)	Z	∂z/∂t	∂² z/∂t ²	t²∂²z/∂t²	1/t
1	-	10	-	-	-	-	-
2	30	5.94	1.781	-	-	-	-
3	60	3.80	1.335	-0.01488	0.000315	1.135073	0.0167
4	90	3.23	1.172	-0.00542	-8.8E-05	-0.7126	0.0111
5	120	2.54	0.931	-0.00806	0.000195	2.814547	0.0083
6	150	2.38	0.865	-0.00219	-9.9E-06	-0.2234	0.0067
7	180	2.20	0.790	-0.00249	-6.7E-06	-0.21736	0.0056
8	210	2.03	0.710	-0.00269	3.66E-05	1.612361	0.0048
9	240	1.94	0.662	-0.0016	3.04E-06	0.175077	0.0042
10	270	1.85	0.617	-0.0015	-8.4E-06	-0.60939	0.0037
11	300	1.76	0.564	-0.00175	-0.00057	-51.1249	0.0033

3.2 Discussion

The dispersion coefficient of Utu Etim-Ekpo River in Akwa Ibom State was obtained from the tracer experiment that was carried out. The dispersion coefficient of the river was computed using analytical and numerical methods as well as empirical formula developed in previous works by various researchers [9,8,6]. The dispersion coefficient obtained from both Leverspiel and Smith method and numerical method were

16.245 m²/s and 14.68 m²/s respectively, while dispersion coefficient obtained from Agunwamba's method gave 19.679 m²/s. The dispersion coefficient from Agunwamba gave a higher value than the other methods due to the fact that the model is best suited for variable distance – variable time method which was the case in Utu Etim-Ekpo River field measurements. It was used as an approximate method for purpose of comparison. The margin of error in the computed dispersion coefficients could be

explained on account of methods of computation using data for constant distance & time in a model of variable distance & time.

A study carried out by Aho et al. [11] gave a dispersion coefficient of $17\text{m}^2/\text{s}$ for Mu river in Benue State, Nigeria with respect to Leverspiel & Smith method. Also, Obi [12] on one dimensional dispersion modeling of natural river channel (with study areas in Nigeria) gave dispersion coefficients ranging from $8.3-24.6\text{ m}^2/\text{s}$, $6.8-13.8\text{ m}^2/\text{s}$, and $13-33.52\text{ m}^2/\text{s}$ for Mmam river, Nwaorie river, and Oji river, respectively. These dispersion coefficients are in agreement with the result of this study with respect to the river geometry which range from 10-20 m in width and 0.3-1.3 m in depth.

The longitudinal dispersion coefficient is affected by the density, channel width, viscosity, flow depth, mean velocity, shear velocity, bed slope, bed roughness, horizontal stream curvature and bed shape factor [13,14]. The laboratory observations have shown that for natural cross-sectional channel geometries, the value of the longitudinal dispersion coefficient can be as much as over 150% greater than the corresponding values obtained for regular channel cross-sections.

4. CONCLUSION

Based on the results of the study, the following conclusion can be drawn:

- The model developed by Agunwamba [6] is in conformity with the method adopted during the sample collection in the river. However, this model is suitable for the determination of dispersion parameters in situation of variable distance & time approach.
- The dispersion coefficient obtained were 16.245 m²/s by Leverspiel & Smith method [9], 14.68 m²/s by Numerical model and 19.679m²/s by Agunwamba method [6].
- The dispersion study is highly essential for the effective management of water quality.
- The contaminant could be discharged into the river either intentionally or accidentally.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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