Asian Journal of Education and Social Studies

Volume 50, Issue 8, Page 215-221, 2024; Article no.AJESS.120203
ISSN: 2581-6268

Division Errors Frequently Made by Students

Suryakanta Behera [a++*] and Pratyush Pattnayak [b++]

[a] Regional Centre Bhubaneswar, IGNOU, India.
[b] Department of Education, Ravenshaw University, India.

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: https://doi.org/10.9734/ajess/2024/v50i81522

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/120203

Original Research Article

Received: 22/05/2024
Accepted: 25/07/2024
Published: 30/07/2024

ABSTRACT

Problems in mathematical problems are common across children of all ages and of all places. This study looks at the common problems students generally face during the mathematical operation of Division and attempts to provide solutions to prevent such problems from taking place further. The study surveyed and analyzed the mistakes of 50 Secondary school students from the district of Cuttack, Odisha in order to isolate the problems students faced during Division. The data was collected through a simple test consisting of a simple long division question and the participants were allowed as much time as they wanted to solve the question. After collecting the responses, the data was analyzed and the responses separated into four different categories. The results showed that a majority I.e. 56% of the students made a mistake in their responses which indicated a lacunae in the base understanding of the process of Division. The mistakes showed a variation in their types, each relating to a different lack of understanding of the process of the mathematical operation.

[++] *Student;*
**Corresponding author: Email: skb@ravenshawuniversity.ac.in;*

***Cite as:** Behera, Suryakanta, and Pratyush Pattnayak. 2024. "Division Errors Frequently Made by Students". Asian Journal of Education and Social Studies 50 (8):215-21. https://doi.org/10.9734/ajess/2024/v50i81522.*

Keywords: Division; mathematics; mistake; model; process.

1. INTRODUCTION

The method of teaching of mathematics also known as the pedagogy of mathematics is as old as humankind itself [1]. While the systematic curriculum oriented teaching may have developed during the modern times, the act of teaching the new generation of students regarding the fundamental manipulation and function of numbers goes as far back as the first appearances of the numbers themselves. Among the various teachings of Mathematics, the four basic operations of Addition, Subtraction, Multiplication and Division are the most common and among them, Division is considered the most complex as it is derived from Subtraction and requires one to memorize the multiplication tables at least. Being the most complex of the operations, Division takes a longer time to teach the students and they commit more mistakes when encountering problems that require division.

While one encounters many common mistakes when teaching children division, the more common among them are the inability to understand that division is spreading an object into equal parts, forgetting/omitting the steps of division, taking the lower number as a result of not knowing their multiplication tables and the inability to progress from Remainders into Decimals due to being taught that the remainder cannot be divided further wheres the decimals are meant to exactly divide the number [2]. Among all of these issues, the omission of certain steps is the most common mistakes that the students make.

2. REVIEW OF LITERATURE

1. The paper studied how the orientation of teachers is towards procedure more than conceptual understanding when it comes to division through a constructivist oriented theoretical framework. The main concept is the study of **Divisibility and its relation to Division, Multiplication, Prime & Composite Numbers, factorization, Divisibility and Prime Decomposition** [3].
2. The paper provides insights into the significant roles of mathematical concepts such as factorization, multiples, prime and composite numbers and prime factorization in the teaching of Division as a basic concept of elementary number theory [4].
3. This paper studied children's comprehension of the concept of Division, by two main methods – Partitive and Quotative, both styles being used under separate circumstances. The study further pressed on the fact that students comprehend the concept of Division better if the problem is presented either pictorially or as a story about sharing of objects within a certain number of people [5].
4. This paper studied how the variation in size of numbers in Division (restricted to simple division problems) causes problems across different ages of students and how students of varied age groups, approach division. The study found that younger students usually performed slower and less accurately compared to older students and relied on the strategy of 'Addition' i.e. adding the divisor repeatedly to get the quotient and older students usually used 'Multiplication' i.e. finding which number multiplied with the divisor would provide the dividend. Furthermore the study tested the prevalence of directly retrieving the answer from memory but observed that division is too unique an operation for its answer to be retrieved from memory [6].

3. METHODOLOGY

3.1 Pedagogy of Division

The teaching of division to children is usually in 2 levels [7]:

I) Primary Level: Dividing solid numbers and deriving quotients and remainders such that no fractional/ decimals have to be used.
II) Higher Level: Dividing any number completely and deriving decimal answers.

3.2 Design

As the investigators have isolated a problem and chosen to describe its occurrence among their sample, the descriptive research design was chosen.

3.3 Sample

50 students of Secondary school level have been selected from the district of Cuttack, Odisha as the sample.

3.4 Data collection Tool

A test consisting of One (1) simple Long Division was used as the tool for collection of data.

3.5 Data Analysis Procedure

The data was analyzed using simple statistical method and graphical method of analysis.

Eucledian Division Lemma: The Euclidean division lemma states, that for any two integers a and , we have two other positive integers q and r such that, $a = bq + r, 0 \leq r < b$. Writing it in numbers, it means that any number a can be written as the sum of the product between two numbers b and q and an indivisible unit left out r. The basis of Euclidean division is Euclid's division algorithm. HCF is the largest number which exactly divides two or more positive integers [8]. That means, on dividing both the integers A and B, the remainder is zero.

Euclidean Division lemma could be applied in order to find the H.C.F. of 2 numbers :

Let us try to find the H.C.F. of Two numbers 64 and 57

64 = (57 × 1) + 7

57 = (7 × 8) + 1 → (The HCF of the numbers 64 and 57 and the quotient)

7 = (1 × 7) + 0

Bottom of the form

(The remainder if 64 is divided by 57)

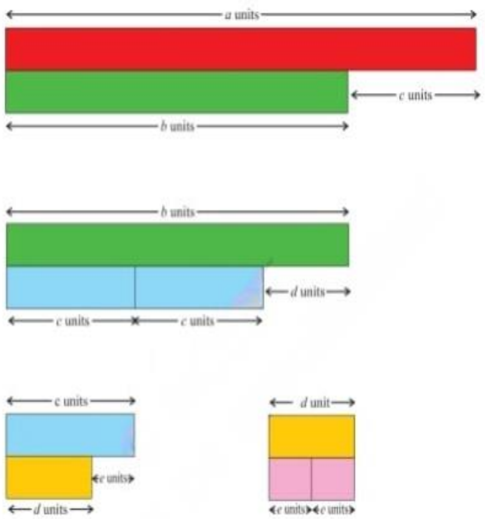

Image 1. H.C.F. of 2 numbers based on Euclidean Division lemma

4. PARTITIVE MODEL OF DIVISION

When talking about division, we can approach it as "Sharing" something between a certain number of receivers [9].

Example - Let 10 oranges be shared equally among 3 friends A, B and C. Thus, each friend shall receive 3 oranges and still there would be a single orange that would remain undivided among them.

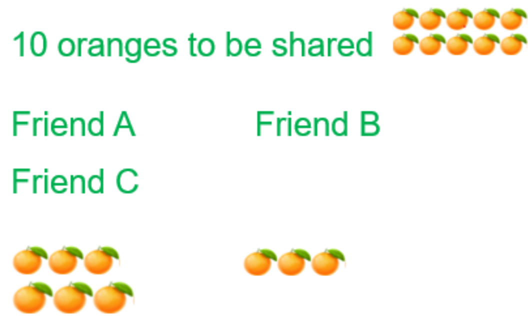

Image 2. A diagrammatic representation would be as follows

Thus, this can be written in numeral form as, "When 10 is Divided by 3, the quotient is 3 and remainder is 1". When taught, the learners will in the beginning start by giving one orange to each receiver and keep going until they reach a stage where the learner falls short of objects to give out to the receivers (while keeping the distribution equal) [10-15].

5. QUOTATIVE MODEL OF DIVISION

Another approach to division that can be taken is that of "how many times can one fit the divisor into the dividend". this approach is known as the Quotative approach and makes use of either **Addition** or **Subtraction (also referred to as chunking)** depending on whether one adds the divisor up to the dividend or subtracts the divisor from the dividends in successive Subtractions.

Example - Let the number 45 be divided by 8 i.e., 45 is to be divided into 8 parts, one can proceed as:-

Addition form	Subtraction form
8 + 8 = 16	45 - 8 = 37
16+8 = 24	37 - 8 = 29
24+8= 32	29 - 8 = 21
32+8 = 40	21 - 8 = 13
40+8 ≠ 45	13 - 8 = 5

Since 8 had to be added 5 times to reach 40 Since 8 had to be subtracted 5 times from 45 and 5 still remained a non reachable amount to reach the number 5 which cannot be further by addition of 8, the Quotient is equal to 5 divided, the quotient is 5 and the remainder too and Remainder is 5.

6. COLLECTION OF DATA

Using a questionnaire, the data of the mathematical competence in Division of various students were collected using the tool. Given below are the results of the data collection process:

Total no. Of data – 50

Table 1. Frequency distribution of variates with cumulative frequency

Variate	Frequency	Cumulative Frequency
Correct answer(370.33)	22	22
Type I Mistake (37.33)	16	38
Type II Mistake (37)	6	44
Type III Mistake (Miscellaneous)	6	50

Correct (370.33) – 22
Type I Mistake (37.33) – 16
Type II Mistake (37) – 6
Type III Mistake (Miscellaneous) - 6

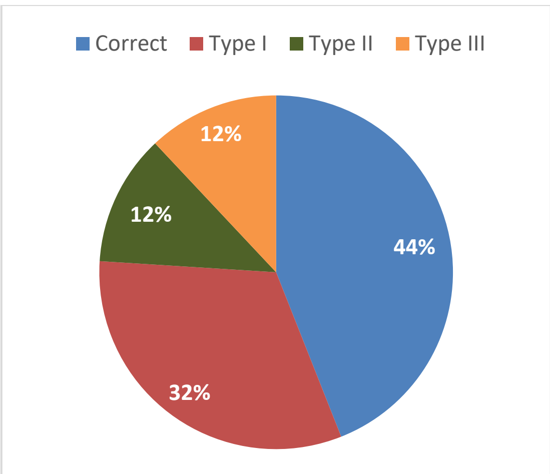

Chart 1. Percentage distribution of variates based on frequency

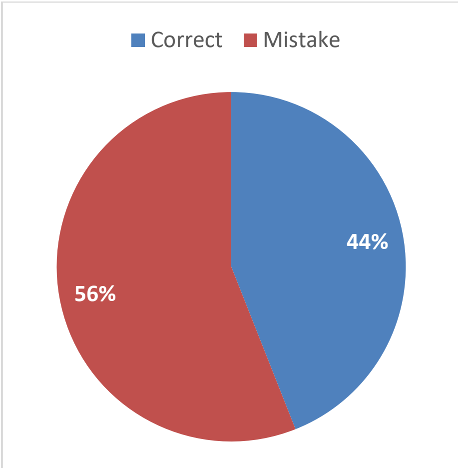

Chart 2. Percentage distribution of correct and mistake

7. DATA ANALYSIS

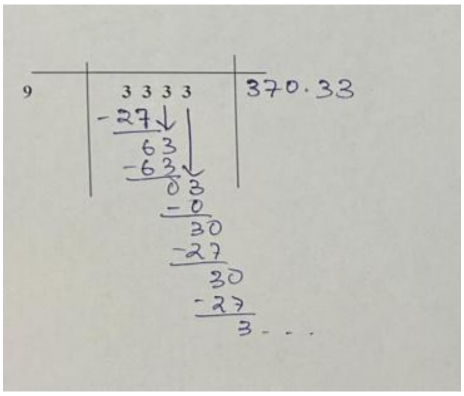

Fig. 1. The correct answer of the division

Correct (22 students, 44% of total respondents)

Division is the process of subtracting the divisor step by step from the dividend until the remainder comes out to be zero (0) or a number while cannot be subtracted from further. In the ‘Fig. 1’, respondent went through the entire process accurately to derive the answer of 370.33….

1. The first step is to divide 33 by 9, which yields the quotient 3 and after subtracting 27 from 33, leaves 6 as the remainder.
2. The next step is to divide 63 by 9, which yields the quotient 7 and after subtracting 63 from 63, the remainder left is 0.
3. The next step is to divide 3 by 9, which yields the quotient 0 and after subtracting 0 from 3, the remainder becomes 3 and the

division can continue only when we make use of a decimal point (.) [16-18].

4. After the application of the decimal point, the division continues forward by dividing 30 by 9 which yields the quotient 3 and remainder 3 and the division continues endlessly. Thus, the final correct answer comes out to be 370.33333……

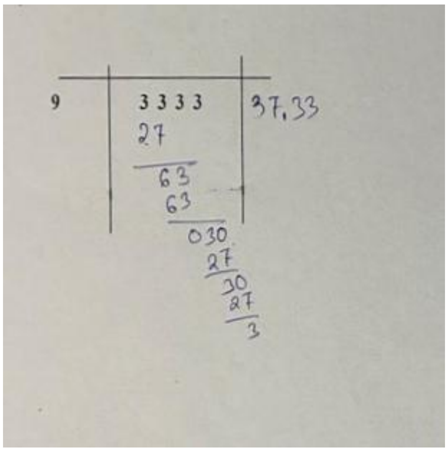

Fig. 2. Type – I mistake of the division

Mistake, Type - I (16 students, 32% of total respondents)

In 'Fig. 2', the respondent went through half the process correctly (Until step 2) but the mistake was committed in step 3. In this instance, the respondent moved on to dividing by the decimal point without dividing the last remaining 3 by 9 and thus, the answer was half-correct, I.e., 37.3333……. or $37.\overline{3}$. This is the mistake with the most frequent occurrence which shows that learners move on to dividing by the decimal without actually completing the final step in the division.

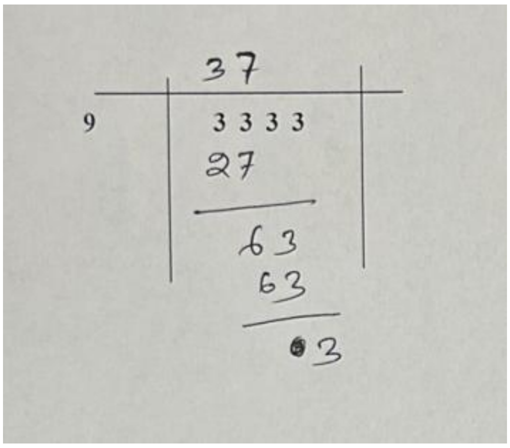

Fig. 3. Type – II mistake of the division

Mistake, Type II (6 students, 12% of total respondents)

In 'Fig. 3', the respondent went through steps 1 and 2 correctly but the mistake occurred at steps 3 and 4, when the final 3 was left undivided and division by decimal point was not done, thereby deriving the answer of 37. This indicates either negligence or disinterest since secondary high students have already been taught division by decimal points.

Miscellaneous, Type III (6 students, 12% of population)

In this case, the respondents mainly made two kinds of mistakes :-

1) Going through step 1 correctly and committing a mistake in step 2 and leaving the operation incomplete.
2) Going through step 1 incorrectly and starting the division with either 2 or 4 as quotient.

8. DISCUSSION

After surveying 50 respondents, the trends of the tests revealed some common findings. The most frequent mistake was the omission of the final step of division which requires the number to be divided with 0 as the quotient before proceeding to division by the decimal point. 44% of the respondents gave the accurate response and 12% of the respondents either showed gross disinterest in the problem or were plain negligent and the mistakes were various ranging from - leaving the division incomplete to dividing with the wrong quotient.

9. CONCLUSION

This study was an attempt to isolate the fundamental problems students face while engaging in the process of division which often goes on to carry over to the future and gets translated to reliance on a calculator even for simple day to day applications of mathematics. Of the three types of mistakes committed, the most frequent was the omission of the step of division with 0 as quotient. This shows that the students do know the overall process of division but the problems lie somewhere in the steps This phenomenon could not be attributed to a rush in solving the problems since the respondents were allowed as much time as they wanted and only willing participants were selected. A logical inference could be that students do not check their divisions by multiplying the quotient with the divisor to see if the result is same as the dividend or somewhere close to it. The Mistakes committed by students were indicative of the lack of clarity in the fundamental operation of division and could be linked to a lack of proper practices in approaching divisions and then checking them which leads one to think that the teaching methods used to teach them were insufficient. In order to make sure that students do not commit such mistakes, we must make sure that the teacher's take more stringent measures to teach and practice problems on division.

10. SIGNIFICANCE

The study showed lack of fundamental understanding in one of the basic mathematical operators and thus, it is important that teachers take note of the findings to avoid future problems while teaching division to students. Based on the findings, this study shall :-

1) Help students avoid dependence on electronic devices to perform simple calculations.
2) Help students learn division in an accurate way and make the concept more appealing to them.
3) Help teachers recognize the mistakes students commit and take steps to combat them.

DISCLAIMER (ARTIFICIAL INTELLIGENCE)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscripts.

CONSENT

As per international standard or university standard, respondents' written consent has been collected and preserved by the author(s).

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Peer-review history:
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