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QCD GHOST DARK ENERGY IN FRACTAL COSMOLOGY

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Abstract. We discuss the interacting QCD ghost dark energy with cold dark matter in the framework of Fractal cosmology. We investigate the cosmological parameters such as Hubble parameter, deceleration parameter and equation of state. We also discuss the physical significance of various cosmological planes like $\omega_D - \omega'_D$ and state-finder. At the end, it is observed that all the results are compatible with observational data.

AMS Mathematics Subject Classification: 83C05.
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1. Introduction

The problem of accelerated expansion is a critical topic in cosmology since its discovery [1]. The main cause of accelerated expansion of the universe is a unknown force so-called dark energy (DE). To explain the nature of DE many cosmologists have proposed many models and theories. Many DE theories for dynamical DE scenario have been proposed to interpret the nature of accelerating universe. A number of DE models have been discussed in this context by many cosmologists. Cosmological constant Λ (ΛCDM) [2] is the simplest candidate for DE (has a constant energy and pressure with constant equation of state). But this model has faced two major problems, cosmic coincidence and fine tuning [2].
In order to describe accelerated expansion phenomenon, two different approaches has been adopted. One is the proposal of various dynamical DE models such as family of chaplygin gas [3], holographic [4, 5], new agegraphic [6], polytropic

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gas [7], pilgrim [8, 9, 10] DE models etc. A second approach for understanding this strange component of the universe is modifying the standard theories of gravity, namely, General Relativity (GR) or Teleparallel Theory Equivalent to GR (TEGR). Several modified theories of gravity are $f(R)$, $f(T)$ [11, 12, 13, 14, 15, 16], $f(R,\mathcal{T})$ [17, 18], $f(G)$ [19, 20, 21, 22, 23] (where R is the curvature scalar, T denotes the torsion scalar, $\mathcal{T}$ is the trace of the energy momentum tensor and G is the invariant of Gauss-Bonnet defined as $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$). For clear review of DE models and modified theories of gravity, see the reference [24].

We arrange the paper as follow: Section **2** describes the basic equations of fractal cosmology. In section **3**, we discuss the cosmological parameters (Hubble, Deceleration, EoS) and planes ($\omega_D - \omega'_D$, state-finder). In the last section, we conclude our results.

2. Fractal Cosmology

According to Einstein gravity in a fractal space-time, the total action is [25, 26]

$$S = S_G + S_m, \tag{1}$$

where S_G is the gravitational part of the action and can be defined as

$$S_G = \frac{1}{16\pi G}\int d\varrho(x)\sqrt{-g}\big(R - 2\Lambda - \omega\partial_\mu\nu\partial^\mu\nu\big), \tag{2}$$

and S_m is the matter part of the action is

$$S_m = \int d\varrho(x)\sqrt{-g}\mathrm{L}_m. \tag{3}$$

Where g is the determinant of the metric (dimensionless) $g_{\mu\nu}$, Λ is the cosmological constant and R is the Ricci scalar, ν and ω are the fractional function and fractal parameter respectively, while the standard measure d^4x is replaced with a Lebesgue-Stieltjes measure $d\varrho(x)$. The Friedmann equation in fractal universe can be obtained after variation of Eq.(1) with respect to the $g_{\mu\nu}$ as

$$H^2 + H\frac{\dot{\nu}}{\nu} - \frac{\omega}{6}\dot{\nu}^2 = \frac{1}{3}(\rho_{de} + \rho_m) + \frac{\Lambda}{3}. \tag{4}$$

Here H denotes the Hubble parameter ($H = \frac{\dot{a}}{a}$), ρ_{cdm} and ρ_{de} are the energy densities due to CDM and DE and $p = p_{de}$ is the pressure of DE. k is the curvature constant with different values of $k = 0, +1, -1$ described as a flat closed and open universe respectively. Λ is the cosmological constant. The continuity equations in fractal universe are given by

$$\dot{\rho_m} + (3H + \frac{\dot{\nu}}{\nu})\rho_m = Q, \tag{5}$$

$$\dot{\rho_{de}} + (3H + \frac{\dot{\nu}}{\nu})(\rho_{de} + p_{de}) = -Q. \tag{6}$$

Where Q describes as the interaction term between DE and CDM with $Q = 3b^2H\rho_m$ and b^2 is a coupling constant.

By assuming a timelike fractal profile $\nu = a^{-\gamma}$ (where γ is the constant), the Friedmann equation becomes

$$H^2(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}) = \frac{1}{3}(\rho_{de}+\rho_m), \tag{7}$$

and the continuity equations can be written as

$$\dot{\rho_m} = (3b^2-3+\gamma)\rho_{m0}a^{(3b^2-3+\gamma)}H. \tag{8}$$

$$\dot{\rho}_{de} + H(3-\gamma)(\rho_{de}+p_{de}) = -3b^2H\rho_{m0}a^{(3b^2-3+\gamma)}. \tag{9}$$

Where ρ_{m0} is the integrating constant.

3. QCD Ghost Dark Energy

Recent observations have been proved that Veneziane ghost of chromodynamics QCD is a good model and helps to solve the U(1) problem [27]. Veneziane ghost DE model contribute to the vacuum energy and proportional to $\Lambda^3_{QCD}H$ (smallest QCD scale), where H is the Hubble parameter and Λ_{QCD} is the QCD mass scale. GDE is defined as [28, 29, 30, 31, 32] $\rho_{de} = \alpha H$. This model is discussed for many cosmological parameter theories and observational schemes. Later on, it has been discussed in the form $H + O(H^2)$ [33] of Veneziane ghost in QCD has enough vacuum energy by which the early evolution of the universe is explained (with the help of H^2) [34]. This model is called generalized ghost DE model (GGDE). Garcia-Salcedo has proposed a new version of GGDE called QCD ghost DE model which depends on the radius of trapping horizon. For flat universe, it is defined as

$$\rho_{de} = \frac{\alpha(1-\epsilon)}{\tilde{r}_T}, \tag{10}$$

where α is numerical constant, $\epsilon = \frac{\dot{\tilde{r}}_T}{2H\tilde{r}_T}$ and $\tilde{r}_T = \frac{1}{H}$. Using these values in Eq.(10), we get

$$\rho_{de} = \alpha(1+\frac{\dot{H}}{2H^2})H. \tag{11}$$

3.1. Hubble Parameter. By using Eqs.(7) and (11), we get the Hubble parameter in the form

$$\frac{\dot{H}}{H^2} = \frac{6}{\alpha}\left(H(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}) - \frac{1}{3H}\rho_{m0}a^{(3b^2-3+\gamma)}\right) - 2. \tag{12}$$

We calculate the Hubble parameter numerically and its plot against the redshift parameter $(1+z)$ with the constant parametric values as shown in figure **1**. We take $d^2 = 0.2, 0.3, 0.4$, $\alpha = 0.5$, $\gamma = 0.1$ and $b^2 = 0.2$. It can be observed from Figure **1** that Hubble parameter corresponds to future day observation of the universe.

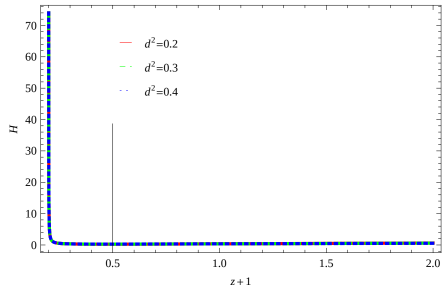

FIGURE 1. Plots of H versus $1 + z$ for QCD ghost DE model in fractal Cosmology.

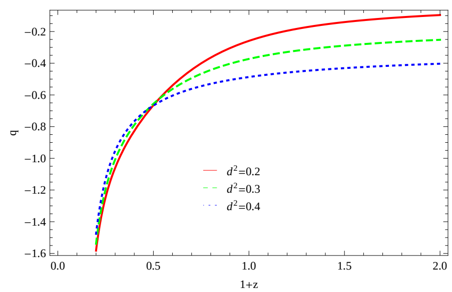

FIGURE 2. Plots of q versus $1 + z$ for QCD ghost DE model in fractal cosmology.

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3.2. Deceleration Parameter. The deceleration parameter is denoted by q. This parameter tells us the transaction phase of the universe, either accelerating ($-1 \leq q < 0$) or decelerating ($q \geq 0$). Its mathematical form is

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2}. \tag{13}$$

After solving the Eqs. (13) and (12), the deceleration parameter is

$$q = -1 - \frac{6}{\alpha}\left(H(1 - \gamma - \frac{\gamma^2\omega a^{-2\gamma}}{6}) - \frac{1}{3H}\rho_{m0}a^{(3b^2-3+\gamma)}\right) + 2. \tag{14}$$

It is cleared from figure **2** that the deceleration parameter corresponds to acceleration expansion of the universe.

3.3. Equation of State Parameter. This parameter can be obtained by using Eqs.(9) and (12) as follows

$$\omega_{de} = -1 - 2\Bigg(\alpha(3-\gamma)\Bigg(2H^2 + \Bigg(\frac{6}{\alpha}\Bigg(H^3\Bigg(1 - \gamma - \frac{\gamma^2\omega a^{-2\gamma}}{6}\Bigg) - \frac{H}{3}\rho_{m0}$$

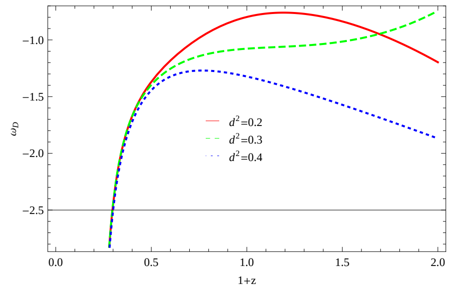

FIGURE 3. Plots of ω_D versus $1+z$ for QCD ghost DE model in fractal Cosmology.

$$
\begin{aligned}
&\times \quad a^{(3b^2-3+\gamma)}\Big)-2H^2\Big)\Big)\Big)^{-1}\Big((3-\gamma)H\rho_{m_0}a^{(3b^2-3+\gamma)}+\gamma^3\omega a^{-2\gamma}H^3\\
&+ \quad \Big(\frac{6}{\alpha}\Big(H^3\Big(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}\Big)-\frac{H}{3}\rho_{m0}a^{(3b^2-3+\gamma)}\Big)-2H^2\Big)\Big(9H\\
&\times \quad \Big(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}\Big)-\rho_{m0}a^{(3b^2-3+\gamma)}\Big)-\frac{\alpha}{2}\Big(\frac{6}{\alpha}\Big(H^3\Big(1-\gamma-\frac{1}{6}\\
&\times \quad \gamma^2\omega a^{-2\gamma}\Big)-\frac{H}{3}\rho_{m0}a^{(3b^2-3+\gamma)}\Big)-2H^2\Big)^2\Big).
\end{aligned}
\tag{15}
$$

The plot of EoS versus redshift parameter is shown in Figure **3**. The EoS parameter behaves quintom-like nature for the interacting case $d^2 = 0.2$. For $d^2 = 0.3$, EoS parameter starts from phantom region and goes towards quintessence region of the universe. However, it remains in the phantom region for $d^2 = 0.4$.

3.4. $\omega_D - \omega'_D$ **plane.** The $\omega_D - \omega'_D$ plane characterize thawing as well as the freezing region of universe, i.e., when $\omega_D < 0$ and $\omega'_D > 0$ then the plane corresponding to thawing region. But if both ω_D and ω'_D are negative, then this plane provides freezing region. Caldwell and Linder [35] discover this method. By taking derivative of Eq.(15), we obtain

$$
\begin{aligned}
\omega_D{}' \quad &= \quad -\frac{2}{H}\Big(6(3-\gamma)\Big(H^3(1-\gamma-\frac{1}{6}\gamma^2\omega a^{-2\gamma})-\frac{H}{3}\rho_{m0}a^{(3b^2-3+\gamma)}\Big)\Big)^{-1}\Big((3\\
&- \quad \gamma)(3b^2-3+\gamma)\rho_{m0}a^{(3b^2-3+\gamma)}H^2-2\gamma^4\omega a^{-2\gamma}H^4+\Big(\frac{6}{\alpha}\Big(H^3(1-\gamma\\
&- \quad \frac{\gamma^2\omega a^{-2\gamma}}{6})-\frac{H}{3}\rho_{m0}a^{(3b^2-3+\gamma)}\Big)-2H^2\Big)\Big((3-\gamma)a^{(3b^2-3+\gamma)}\rho_{m0}+3\\
&\times \quad \gamma^3\omega a^{-2\gamma}H^2+\Big(\frac{6}{\alpha}\Big(H^3\Big(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}\Big)-\frac{H}{3}a^{(3b^2-3+\gamma)}\rho_{mo}\Big)-2
\end{aligned}
$$

$$
\begin{aligned}
\times\quad & H^2\Big)\Big(9\Big(1-\gamma-\frac{1}{6}\gamma^2\omega a^{-2\gamma}\Big)+\frac{1}{H^2}\rho_{m0}a^{(3b^2-3+\gamma)}\Big)+3\gamma^3\omega a^{-2\gamma}H^2\\
-\quad & (3b^2-3+\gamma)\rho_{m0}a^{(3b^2-3+\gamma)}+\frac{\alpha}{H^3}\Big(\frac{6}{\alpha}\Big(H^3\Big(1-\gamma-\frac{1}{6}\gamma^2\omega a^{-2\gamma}\Big)-\frac{1}{3}\\
\times\quad & \rho_{m0}a^{(3b^2-3+\gamma)}H\Big)-2H^2\Big)^2-\frac{\alpha}{H^2}\Big(\Big(\frac{18}{\alpha}H^2\Big(1-\gamma-\frac{1}{6}\gamma^2\omega a^{-2\gamma}\Big)-\frac{2}{\alpha}\\
\times\quad & \rho_{m0}a^{(3b^2-3+\gamma)}-2H\Big)\Big(\frac{6}{\alpha}\Big(H^3\Big(1-\gamma-\frac{1}{6}\gamma^2\omega a^{-2\gamma}\Big)-\frac{H}{3}a^{(3b^2-3+\gamma)}\\
\times\quad & \rho_{m0}\Big)-2H^2\Big)+\frac{2}{\alpha}\gamma^3\omega a^{-2\gamma}H^4-\frac{2}{\alpha}(3b^2-3+\gamma)a^{(3b^2-3+\gamma)}\rho_{m0}H^2\Big)\\
\times\quad & \Big)+\Big(\Big(\frac{18}{\alpha}H^2\Big(1-\gamma-\frac{1}{6}\gamma^2\omega a^{-2\gamma}\Big)-\frac{2}{\alpha}\rho_{m0}a^{(3b^2-3+\gamma)}-2H\Big)\Big(\frac{6}{\alpha}\Big(\\
\times\quad & H^3\Big(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}\Big)-\frac{H}{3}\rho_{m0}a^{(3b^2-3+\gamma)}\Big)-2H^2\Big)+\frac{2}{\alpha}\gamma^3a^{-2\gamma}H^4\\
\times\quad & \omega-\frac{2}{\alpha}(3b^2-3+\gamma)\rho_{m0}a^{(3b^2-3+\gamma)}H^2\Big)\Big(9H\Big(1-\gamma-\frac{1}{6}\gamma^2a^{-2\gamma}\omega\Big)-\\
\times\quad & \rho_{m0}a^{(3b^2-3+\gamma)}\Big)\Big)+\frac{2}{H}\Big(\frac{36}{\alpha}(3-\gamma)\Big(H^3\Big(1-\gamma-\frac{1}{6}\gamma^2\omega a^{-2\gamma}\Big)-\frac{H}{3}\\
\times\quad & \rho_{m0}a^{(3b^2-3+\gamma)}\Big)^2\Big)^{-1}\Big((3-\gamma)\rho_{m0}a^{(3b^2-3+\gamma)}H+\Big(9H\Big(1-\gamma-\frac{1}{6}\gamma^2\\
\times\quad & \omega a^{-2\gamma}\Big)-\frac{1}{H}\rho_{m0}a^{(3b^2-3+\gamma)}\Big)(\frac{6}{\alpha}\Big(H^3\Big(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}\Big)-\frac{H}{3}\rho_{m0}\\
\times\quad & a^{(3b^2-3+\gamma)}\Big)-2H^2)+\gamma^3\omega a^{-2\gamma}H^3-\frac{\alpha}{2H^2}(\frac{6}{\alpha}\Big(H^3\Big(1-\gamma-\frac{1}{6}\gamma^2a^{-2\gamma}\\
\times\quad & \omega\Big)-\frac{H}{3}\rho_{m0}a^{(3b^2-3+\gamma)}\Big)-2H^2)^2\Big)\Big(4H\Big(\frac{6}{\alpha}\Big(H^3\Big(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}\Big)\\
-\quad & \frac{H}{3}\rho_{m0}a^{(3b^2-3+\gamma)}\Big)-2H^2\Big)+\Big(\Big(\frac{18}{\alpha}H^2\Big(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}\Big)-\frac{2}{\alpha}\rho_{m0}\\
\times\quad & a^{(3b^2-3+\gamma)}-2H\Big)\Big(\frac{6}{\alpha}\Big(H^3\Big(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}\Big)-\frac{H}{3}\rho_{m0}a^{(3b^2-3+\gamma)}\Big)\\
-\quad & 2H^2\Big)+\frac{2}{\alpha}\gamma^3\omega a^{-2\gamma}H^4-\frac{2}{\alpha}(3b^2-3+\gamma)\rho_{m0}a^{(3b^2-3+\gamma)}H^2\Big)\Big).
\end{aligned}
\tag{16}
$$

The behavior of $\omega_D-\omega'_D$ can be observed from Figure **4** which exhibits the freezing region.

3.5. $r-s$ **plane.** With the help of this plane, we can identify different DE models. Trajectories of different DE models have different ranges in this plane. For example, $\{r,s\}=\{1,0\}$ corresponds to ΛCDM model, $\{r,s\}=\{1,1\}$ shows

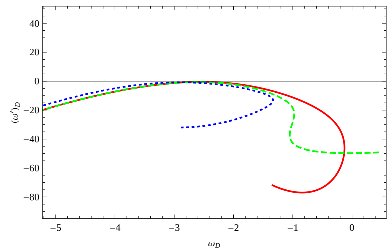

FIGURE 4. Plots of $\omega_D - \omega'_D$ for QCD ghost DE model in fractal cosmology.

CDM limit, $\{s > 0, r < 1\}$ shows phantom and quintessence while $\{s < 0, r > 1\}$ denotes chaplygin gas region. Mathematical form of state-finder parameters are given by [36]

$$r = 1 + \frac{3\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \ and \ s = \frac{r-1}{3(q-\frac{1}{2})}. \tag{17}$$

To obtain the values of $r - s$ plane we substitute Eq. (12) and (14) in (17),

$$\begin{aligned} r &= 1 + \left(\frac{6}{\alpha}\left(H^3(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}) - \frac{H}{3}\rho_{m0}a^{(3b^2-3+\gamma)}\right) - 2H^2\right) \\ &\times \left(3 + \frac{9H}{\alpha}(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}) - \frac{2}{H\alpha}\rho_{m_0}a^{(3b^2-3+\gamma)} - 2\right) + \frac{2H^4}{\alpha} \\ &\times \gamma^3\omega a^{-2\gamma} - \frac{2H^2}{\alpha}(3b^2-3+\gamma)\rho_{m0}a^{(3b^2-3+\gamma)} \end{aligned} \tag{18}$$

$$\begin{aligned} s &= \frac{1}{3}\left(-\frac{3}{2} - \frac{6}{\alpha}\left(H(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}) - \frac{1}{3H}\rho_{m0}a^{(3b^2-3+\gamma)}\right) + 2\right)^{-1} \\ &\times \left(\left(\frac{6}{\alpha}\left(H^3(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}) - \frac{H}{3}\rho_{m0}a^{(3b^2-3+\gamma)}\right) - 2H^2\right)\right. \\ &\times \left(3 + \frac{9H}{\alpha}(1-\gamma-\frac{\gamma^2\omega a^{-2\gamma}}{6}) - \frac{2}{H\alpha}\rho_{m0}a^{(3b^2-3+\gamma)} - 2\right) + \frac{2H^4}{\alpha} \\ &\times \left.\gamma^3\omega a^{-2\gamma} - \frac{2H^2}{\alpha}(3b^2-3+\gamma)\rho_{m0}a^{(3b^2-3+\gamma)}\right). \end{aligned} \tag{19}$$

The plane of this model is given in Figure **5**. The $r - s$ plane for this model shows the Chaplygin gas behavior as well as ΛCDM model.

4. Concluding Remarks

We have investigated the physical significance of QCD ghost DE model in fractal universe by developing various cosmological parameters as well as cosmological

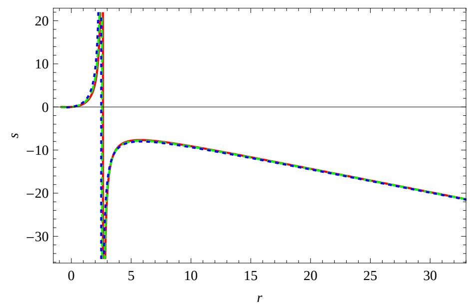

FIGURE 5. Plots of $r-s$ for QCD ghost DE model in fractal Cosmology.

planes. These parameters as well as planes shamefully explain the current cosmic acceleration.

Competing Interests

The author(s) do not have any competing interests in the manuscript.

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