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Boolean Product of Zero-one Matrices Application to Truth Values of Logical Connectives of Several Propositions

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

In this paper, the Boolean product of zero-one matrices are applied to obtain the truth values of several propositions with logical connectives. The propositional matrices are given in relation to the matrix algebric properties. We state and prove a theorem for which a given conditional connetives is a tautology.

Keywords: Boolean product; logical connectives; propositional matrix; tautology and truth values.

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1 Introduction

Boolean algebra provides the operator and rules for working with the set of element 0 and 1 whose product with respect to the binary operations of AND and OR denoted by $\wedge$ and $\vee$ respectively are defined between only pairs of elements as

$$a_{ij} = \begin{cases} 1 & \text{if} \quad i = j = 1 \\ 0 & \text{otherwise} \end{cases} \tag{1.1}$$

and

$$b_{ij} = \begin{cases} 1 & \text{if} \quad i \text{ or } j = 1 \\ 0 & \text{otherwise} \end{cases} \tag{1.2}$$

various application abound in real life associated with the used of Boolean product to determine the truth outcomes and values of event [1, 2]. The study of combinatorial circuits for current flow, computer informatics and logical connectives truth values table are prominent in their usage [3, 4, 5].

Research interest is to obtain a propositional matrix arising from the truth value or symbolic tables of proposition and thereby using the matrix properties to determine the truth value of propositions with logical connectives. So far researches and illustration known had been on two propositions p and q [4]. In this work therefore, the Boolean products of zero-one matrix are applied to obtain propositional matrices and their truth values for several propositions with logical connectives also the theorem for which a conditional connectives is a tautology is formulated and proved.

2 Propositional Matrix/Results

Sequel to basic definitions and notations, let p, q and r be propositions with assigned truth values $p = (1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0)$, $q = (1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0)$ and $r = (1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0)$ and $p \wedge q$, $p \vee q$ be as defined AND and OR in symbolic logic respectively.

Definition 2.1. Propositional matrix B for any logical connectives consists of the Boolean product of the symbols of each proposition such that bij represents the product of $i^{th}-$symbol of p and the $j^{th}-$symbol of q

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix} \tag{2.1}$$

Thus the diagonal entries of the matrix B denoted by $T(B) = (b_{11}, \quad b_{22}, \quad b_{33}, \quad b_{44})$ represent the truth values of the components propositions with logical connectives in the event of the propositions, we have that if $p = (1 \quad 1 \quad 0 \quad 0)$ and $q = (1 \quad 0 \quad 1 \quad 0)$ such that

$$\begin{aligned} B_1 = p \wedge q \quad &= \quad (1 \quad 1 \quad 0 \quad 0) \wedge (1 \quad 0 \quad 1 \quad 0) \\ &= \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \tag{2.2}$$

and

$$B_2 = p \vee q = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix} \tag{2.3}$$

then the resulting truth value will be

$$T(B_1) = (1 \quad 0 \quad 0 \quad 0) \tag{2.4}$$

and

$$T(B_2) = (1 \quad 1 \quad 1 \quad 0) \tag{2.5}$$

B_1 and B_2 are called propositional matrices. In this paper, the results above are extended to several propositions, showing the distributive law of disjunction over conjunction with logically equivalent propositions.

Consider the proposition $A_1 = p \wedge q \wedge r$ and $A_2 = p \vee q \vee r$ the truth values of the combinational connectives A_1 and A_2 written $T(A_1)$ and $T(A_2)$ are obtained by applying the algebraic properties of associativity, commutativity and distributive laws of the symbols. Thus for $p = (1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0)$, $q \wedge r = (1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0)$ or $p \wedge q = (1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$, $r = (1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0)$, the propositional matrix for A_1 becomes

$$A_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.6}$$

such that

$$T(A_1) = (1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0) \tag{2.7}$$

Similarily,

$$\begin{aligned} p \vee q &= (1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0) \quad r = (1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0) \\ A_2 &= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \end{aligned} \tag{2.8}$$

and

$$T(A_2) = (1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0) \tag{2.9}$$

Let A_3 be the logical equivalent proposition $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$. Using the propositional matrix of Boolean product, the truth values for the biconditional connectives is derived where $p \vee q = (1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0)$ and

$p \vee r = (1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0)$

$$A_3 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.10}$$

such that

$$T(A_3) = (1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0) \tag{2.11}$$

On the other hand, using $p = (1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0)$ and $q \wedge r = (1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0)$

$$A_3' \quad = \quad \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \tag{2.12}$$

and

$$T(A_3') = (1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0) \tag{2.13}$$

hence $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$.

Lemma 2.1. *: Let p, q and r be propositions, the conditional connectives of the conjunction over the disconjunction of p, q and r is a tautology.*

Proof:

$$\begin{aligned} \text{Let} \quad (p \wedge q \wedge r) \quad &\rightarrow \quad (p \vee q \vee r) \\ &\Leftrightarrow \quad -(p \wedge q \wedge r) \vee (p \vee q \vee r) \\ &\Leftrightarrow \quad (-p \vee -q \vee -r) \vee (p \vee q \vee r) \\ &\Leftrightarrow \quad (-p \vee p) \vee (-q \vee q) \vee (-r \vee r) \\ &\Leftrightarrow \quad T \vee T \vee T \\ &\Leftrightarrow \quad T. \end{aligned}$$

Using propositional matrices of A_1 and A_2, let $A_1 \rightarrow A_2$, then $T(A_1) \rightarrow T(A_2)$ where $T(A_1)$ and $T(A_2)$ are as in equation (2.7) and (2.9). The resulting truth values is $(1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad) = T$.

Theorem 2.2. *Let $P_i, i = 1, 2, ..., n$ be propositions such that $\prod_i \wedge P_i$ and $\prod_i \vee P_i$ denote the conjunction and disconjunction of several propositions. Then $\prod_i \wedge P_i \rightarrow \prod_i \vee P_i$ is a tautology.*

Proof: Let $B(b_{ij})$ and $C(c_{ij})$ be propositional matrices of the conjunction and disconjunction defined as

$$B = b_{ij} = \begin{cases} 1 & \text{if} \qquad P_i \cap P_j = 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$C = c_{ij} = \begin{cases} 0 & \text{if} \quad P_i \cap P_j = 0 \\ 1 & \text{otherwise} \quad i, j = 1, 2, ..., n \end{cases}$$

where $P_i \cap P_j$ indicates the intersection of the truth values of P_i and P_j at the $(i, j)^{th}$ symbols. Then $T(B) = (1 \quad 0 \quad 0 \quad . \quad . \quad . \quad 0)$ and $T(C) = (1 \quad 1 \quad 1 \quad . \quad . \quad . \quad 1 \quad 0)$ such that the $2^n \times 2^n$ propositional matrix $A(a_{ij})$ becomes a unitary matrix, hence $T(a_{11} \quad a_{12} \quad . \quad . \quad . \quad a_{1n}) = (1 \quad 1 \quad 1 \quad . \quad . \quad . \quad 1 \quad 1)$.

3 Conclusion

The propositional matrix and its truth values presented and the theorem given, lay a foundation for further research and application in any area of relevances.

Competing Interests

Author has declared that no competing interests exist.

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