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Dynamic Biplot. Evolution of the Economic Freedom in the European Union

Jaime Egido[1*] and Purificación Galindo[1]

[1]Department of Statistic, University of Salamanca, C/ Alfonso X El Sabio s/n, 37007 Salamanca, Spain.

Authors' contributions

This work was carried out in collaboration between both authors. Author JE designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Authors JE and PG managed the analyses of the study and literature searches. Both authors read and approved the final manuscript.

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ABSTRACT

This paper presents a new method called 'Dynamic Biplot' that applies to the representation of 3-way data. The Dynamic Biplot method is undertaken in two stages: First, it carries out a biplot analysis of the situation that is considered more representative in our analysis, the reference situation (static analysis); in the second stage, the rest of the situations are represented onto the analysis (dynamic analysis), obtaining trajectories of individuals and variables. The Dynamic Biplot method is used to analyse the evolution of the economic freedom in the European Union. The main results are that the countries outside the euro zone enjoy greater economic freedom and the *Rule of law area* (*Property Rights* and *Freedom from Corruption*) are the ones that better discriminate EU countries. To provide operational support to the Dynamic Biplot, it has developed a package in R and it has been applied to describe the evolution of the economic freedom in the EU.

Keywords: Dynamic biplot; economic freedom; European Union; multivariate; three-way.

**Corresponding author: E-mail: jegido@usal.es;*

1. INTRODUCTION AND BIPLOT REVIEW

Among the various methods used for multivariate data analysis, Gabriel [1] Biplot methods are capable of representing more than two variables [2] using an approximation from the n-dimensional space to another of lower dimension, generally two. This method was introduced by Gabriel [1] in the context of principal component analysis (PCA). Specifically, the biplot is a joint graphical representation, in a low dimensional Euclidean space (usually a plane), of a multivariate data matrix by markers for its rows and columns, chosen in such a way that the inner (or scalar) product represents the elements of the data matrix. Due to the properties of the inner product, the biplot is a powerful data visualization tool, which can be viewed as a multivariate extension of the scatterplot.

The method used to obtain a reduced rank approximation of a matrix is the one proposed by Eckart and Young [3] and has been used by Householder and Young [4], Gabriel [1], Greenacre [5] and other authors. An algorithm to compute the decomposition is shown in Golub and Reinsch [6]. Galindo [7] developed the HJ-Biplot method to get high quality of representation for rows and columns simultaneously. These factorizations have formed the basis for the development of multiple approaches. The treatment of multiple tables with biplot methods has been discussed in Meta-Biplot [8], Canonical STATIS - CANOSTATIS [9] which generalizes discriminant analysis with STATIS.

The data that have been processed have a 3-way structure. In the literature, the treatment of a data cube has been performed in different ways: concatenating the matrices [10,11], comparing the main components of the groups [12,13], Lavit et al. [14] propose the diagonalization of a matrix, called 'common object' of the nature of the groups, Flury [15] considering different models of similarity between the components.

One of the most popular trends is based on obtaining a consensus subspace for all arrays, with different solutions like, e.g., STATIS and Dual STATIS [14,16,17] for the same set of individuals or variables respectively, Pagès and Escoufier [18] proposed the Multiple Factor Analysis - MFA for the same individuals in several sets of variables, Chessel and Hanafi [19] proposed the Multiple Co-Inertia Analysis – MCOIA when the number of samples is low compared to the number of variables, and Dazy and Barzic [20] use Double Principal Component Analysis – DPCA for these individuals and the same variables. STATIS methods, have had many contributions; as e.g., Jaffrenou [21] use the Partial Triadic Analysis - PTA for the same variables measured on the same observations; Simier et al. [22] proposed the STATICO method; Suazay et al. [23] presented a PTA on series of cross product tables, external STATIS – (K+1)-STATIS with one external data set; Abdi et al. [24] used DISTATIS for distance matrices; Sabatier and Vivien [25] implement STATIS-4 for more than two sets of data; Vivien and Sune [26] used Double-STATIS (DO-ACT) to (K+1)-STATIS for two sets of data; Thioulouse [27] developed COVSTATIS for covariance matrices and COSTATIS, a co-inertia analysis of the compromises computed by the PTA; Bâenassâeni and Bennani Dosse [28] used Power STATIS-ACT to set weights to the matrices; Corrales and Rodriguez [29] developed INTERSTATIS for interval valued data. A review of the STATIS methods is shown in Abdi et al. [30].

This paper proposes the use of a new technique for representing 3-way data: the Dynamic Biplot. It is given that the term Dynamic Biplot has been already used by Sparks et al. [31] for quality control processing, considering a 2-way array. To provide operational support to the theory of Dynamic Biplot, it has developed a package in R with a graphic user interface (GUI) that has been called 'dynBiplotGUI'.

The structure of this paper is as follows. First it is performed a review of the Biplot methods. In the next two sections it makes the working approach and it develops the Dynamic Biplot. In another section it presents the R package developed to support Dynamic Biplot. Finally, it presents data on Economic Freedom in the European Union and the results obtained by applying the technique exposed.

2. WORKING APPROACH

In Biplot methods, variables are generally represented by vectors where the direction responds to the one that best represents the change in each variable. There are infinite possible factorizations in low-level decompositions to produce rows and columns markers: depending on the chosen one, different properties are obtained. Among the biplot methods developed by Gabriel [1], GH-Biplot provides high-quality for columns and JK-Biplot representing rows. Galindo [7] developed the HJ-Biplot which performs high quality for rows and columns simultaneously.

The approach presented in this paper is radically different to the method used by Sparks et al. [31] for the Dynamic Biplot, which is a table of 2-way data where one adds the number of cases that occur. It aims to study the relationship of a set of data that occurs more than once, i.e. of a 3-way dataset. It is common, when working with economic data, to study a certain magnitude (GDP, production ...) concerning different entities (countries, companies...) in repetitive situations (year, production cycles...).

The STATIS method puts its focus on the analysis of individuals while in the Dual STATIS focus are the variables [14,17]. Both methods treat the data in three steps. (i) Computing the interstructure of the data matrices, diagonalizing a matrix of scalar products between tables. (ii) Computing the compromise matrix, a linear combination of the initial tables with the aim of constructing a mean table of maximum inertia. (iii) Performing the analysis of the intrastructure and trajectories, summarizing the variability of the tables in comparison to the common structure defined by the compromise matrix.

Compared to STATIS methods that achieve the common part of all matrices and calculate the trajectories of individuals or variables, the Dynamic Biplot method proposed for us starts with the best representation of a given situation and simultaneously obtains both trajectories.

The nomenclature used is: $\boldsymbol{C}$ is a data cube of dimension $n \times p \times q$; the 3rd way are the situations and t is the reference situation; the $n \times p$ data matrix $\boldsymbol{Y}$ represents the baseline t, and its decomposition is $\boldsymbol{Y=AB'=UDV'}$; $\boldsymbol{z}$, $\boldsymbol{x}$ are generic vectors and $\boldsymbol{s}$, $\boldsymbol{o}$, are their projections; $\boldsymbol{J=UD}$, $\boldsymbol{K=V}$, $\boldsymbol{G=U}$, $\boldsymbol{H=VD}$ call to different decompositions of matrix $\boldsymbol{Y}$. $\boldsymbol{Z}$ is a general matrix with elements to be projected and $\boldsymbol{L}$, $\boldsymbol{M}$ corresponding to the row and column projections. The $n \times q$ matrix $\boldsymbol{Z}_j$, contains the values of the variable j in different situations, being $1 \le j \le p$. The $p \times q$ matrix $\boldsymbol{X}_i$, contains values of the individual i in different situations, with $1 \le i \le n$.

3. THE DYNAMIC BIPLOT

Let $\boldsymbol{C}$ be a 3-way data matrix: Individuals are entered in rows, variables in columns, and situations for various occasions; usually correspond to the same subjects and variables but measured at different times. Is set t as the base situation according to which will be represented the dynamism of the rest; t is the situation that wants to analyze and it is considered to have the more explanatory information for our analysis.

The proposed technique, the Dynamic Biplot, performs a multivariate analysis of the baseline using biplot analysis, providing us with the best multivariate information between variables, individuals and relationships between them. The first step corresponds to the static analysis of our procedure, according the biplot criterion. The second step, the dynamic analysis, projects the rest of the situations it wants to analyze on the biplot graph obtained in the previous step. The result is the set of trajectories of individuals and variables on the reference situation biplot.

3.1 Step 1, Static Analysis

Let $\boldsymbol{C}$ be the data matrix, data cube, of dimension $n \times p \times q$ (Fig. 1a). They are extracted from these data those corresponding to a time t, which will be used as a reference, obtaining a matrix $\boldsymbol{Y}$ of dimension $n \times p$. Factorizing matrix $\boldsymbol{Y=AB'}$ through the singular values decomposition proposed by Eckart and Young [3] aiming to achieve a low rank approximation of the matrix in the sense of least squares, is obtains $\boldsymbol{Y=UDV'}$. Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ contain markers of rows and columns respectively. Matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ contain the eigenvectors, and the matrix $\boldsymbol{D}$, the eigenvalues. Depending on the chosen factorization and adapting the nomenclature $\boldsymbol{A}$ and $\boldsymbol{B}$ to the customary name, named $\boldsymbol{J=UD}$, $\boldsymbol{K=V}$, $\boldsymbol{G=U}$, $\boldsymbol{H=VD}$, which creates the different types of biplot: JK-Biplot, GH-Biplot y HJ-Biplot. With markers calculated, it is produced the biplot graph of the reference situation t.

Galindo [7] shows us how the HJ-Biplot methods are interpreted, which is based on simple geometric concepts. Below those basic interpretational rules are presented: (i) The distances among row markers are interpreted as an inverse function of similarities, in such a way that closer markers (objects) are more similar; (ii) The lengths of the column markers (vectors) approximate the standard deviation of the variables; (iii) The cosines of the angles among the column vectors approximate the correlations among variables in such a way that small acute angles are associated with variables with high positive correlations; obtuse angles near to the straight angle are associated with variables with high negative correlations and right angles are associated with non-correlated variables. In the same way, the cosines of the angles among the variable markers and the axes (Principal Components) approximate the correlations between them; (iv) The order of the orthogonal projections of the row markers (points) onto a column marker (vector) approximates the order of the row elements (values) in that column.

3.2 Step 2, Representation of Elements onto the Biplot

The goal in this second step is to project, onto the biplot obtained in the previous step, the rest of situations included in the cube ***C***, retaining structures previously computed in *t*. These data should not be taken into account when calculating the biplot reference markers because they correspond to past situations that do not have to change the context under consideration. However, it is important to represent their positions and evolution in order to observe the dynamic of its evolution. To incorporate new information in the graph, must be found the markers in the reduced dimension space representing the element to be projected. Those markers must be optimal in the sense of least squares. Our proposal works in a similar way as Graffelman and Aluja-Banet [32].

3.2.1 Representation of variables onto the reference biplot

Let ***z*** be a column vector with *n* values of a variable to be incorporated into the biplot already obtained and ***s*** the projection vector as $\hat{z}$ discussed above. The best fitting projection vector to ***z*** will be that minimizing the sum of the squared differences between them. Is obtained vector performing the scalar product between the orthogonal matrix ***A,*** with row markers, and vector ***z***. This vector is optimum if the sum of squared errors is minimum. Calculating and setting to zero the first order derivative, is obtained the solution to the minimization problem:

$$\boldsymbol{s} = (\boldsymbol{A}'\boldsymbol{A})^{-1}\boldsymbol{A}'\boldsymbol{z} \qquad (1)$$

The solution to (1) are the regression coefficients obtained from the regression of the variable to be projected and the columns of the matrix with row markers obtained in the singular value decomposition of the data matrix in the reference situation *t*. As the regression variables are orthogonal, their coefficients are independent.

3.2.2 Representation of individuals onto the reference biplot

If you expect to add the position of an individual, i.e., a row, in the previously calculated biplot, the procedure is similar. Let ***x*** be a column vector with *p* values of the individual to be incorporated and let ***o*** be the point in the biplot plane that represents it. It is assumed that the vector ***x'*** is centered by the reference situation mean vector. Is obtained the $\hat{x}$ vector for the additional point by calculating the scalar product of ***o*** with all the variables' vectors represented in the biplot, which define the rows of the matrix ***B***. This value is optimum if the sum of squared errors is minimum. Calculating and setting to zero the first order derivative is obtained the solution to the minimization problem:

$$\boldsymbol{o} = (\boldsymbol{B}'\boldsymbol{B})^{-1}\boldsymbol{B}'\boldsymbol{x} \qquad (2)$$

The solution to (2) is again the regression coefficients between the point to be projected and the columns of the matrix ***B***. The coefficients are independent since the columns of the matrix ***B*** are orthogonal.

Generalizing the foregoing, if ***Z*** is the matrix with several variables or individuals that it is intended to project in the biplot and ***LM*** is its decomposition, they can be obtained for equations (1) and (2) respectively:

$$\boldsymbol{L}' = (\boldsymbol{A}'\boldsymbol{A})^{-1}\boldsymbol{A}'\boldsymbol{Z};\ \boldsymbol{M}' = (\boldsymbol{B}'\boldsymbol{B})^{-1}\boldsymbol{B}'\boldsymbol{Z}'.$$

3.2.3 Trajectories

Being *t* the reference situation, they can be extracted from the data cube ***C*** as many two-dimensional arrays as variables they have, i.e., *p* matrices of dimension $n \times q$; the scalar *n* corresponds to the number of individuals (rows) of ***Y***, and *q* to the number of values that the variable has had along the *q* situations. Let $\boldsymbol{Z_j}$ be each of the matrices thus obtained, with $1 \le j \le p$, once the data has been centered. On Fig. b, it can be seen the representation of one of the matrices. Let us proceed to project $\boldsymbol{Z_j}$ in the biplot graph obtained in step 1 as seen previously. If are joined each of the points representing the values that the variable has had in different situations, and in the right order, can be obtained the path followed by this variable in relation to the variables and points represented at time *t*. By varying *j* conveniently, is obtained the representation of the trajectories of all variables.

Similarly, once the reference situation *t* has been set in the data cube ***C***, they can get as many two-dimensional arrays as individuals, i.e., *n* matrices of dimension $p \times q$ where the scalar *p* corresponds to the number of variables (columns) of ***Y*** and *q* to the number of values that individuals have had along the *q* situations. Let $\boldsymbol{X_i}$ be each of the matrices thus obtained, with $1 \le i \le n$, once the data have been centered with the value of the mean of the variables. On Fig. 1c it can be seen the representation of one of the matrices. Projecting $\boldsymbol{X_i}$ in the biplot graph obtained in step 1 and joining every point representing the values that an individual has had in different situations, and in the right order, is obtained the path followed by the individual in relation to the variables and points represented by situation *t*. Varying *i* conveniently is obtained the representation of the trajectories of all individuals.

3.2.4 Dynamic biplot properties

The biplot analysis of a reference context has all the properties of its chosen factorization. The correct interpretation will give us knowledge of the static situation *t*. The elements projected in different situations retain properties similar to those of the chosen factorization regarding the reference situation.

4. THE DYNBIPLOT SOFTWARE

To provide operational support to the theory behind the Dynamic Biplot method, it has developed a package in R with a Graphical User Interface (GUI) that it has been called dynBiplotGUI. It covers everything one may need to make and represent the Dynamic Biplot.

Fig. 2. shows the appearance of the program window with the loaded data cube.

The package dynBiplotGUI contains the program dynBiplot, which performs the Dynamic Biplot. All processing is carried out with GUI, and can be done using Spanish, English, French and Portuguese. The outcome is graphical and numerical results of the analysis.

The program dynBiplot contains four panels that allow us to carry out each of the tasks required for the analysis, including guiding the user to minimize errors. The first panel, namely 'Data Panel', allows to load the data onto the module and to prepare its structure depending on whether they are 2-way or 3-way data. The format of the individuals and variables is done in the 'Format Panel'. In the 'Variables Panel' you can be selected items, individuals and variables, which they can be used in the calculation of the model. Finally, the 'Analysis Panel' contains all the necessary options to customize the analysis and the chart.

Once the graph is obtained, it is possible to change the settings and get a new one as many times as necessary, without having to close the program and run it again. On the chart itself, one can make the zoom needed to get more detail.

5. EVOLUTION OF THE ECONOMIC FREEDOM IN THE EUROPEAN UNION

5.1 Introduction

As result of the Economic and Monetary Union (EMU), signed in 1992 by the Maastricht Treaty, the European Union (EU) has a set of economic and monetary policies that apply to its 27 member countries [33]. This treaty gave rise to the single currency, the euro, used by the 17 EU countries forming the 'euro area'. On July 1st, 2013, Croatia has become part of the EU.

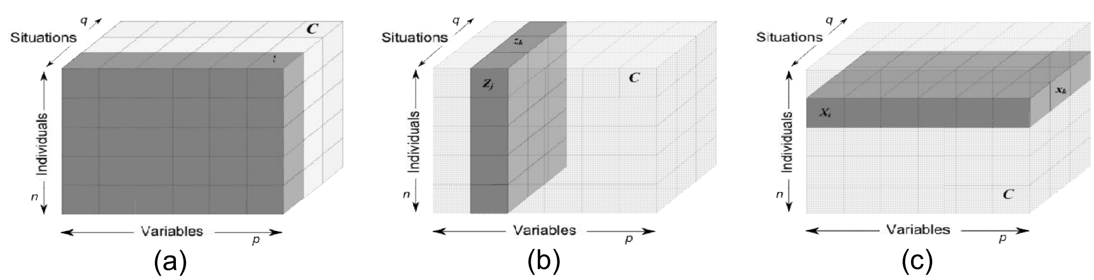

Fig. 1. Data cube representing the reference situation *t* (a); matrix situations for variable y_j (b) and the individual y_i (c)

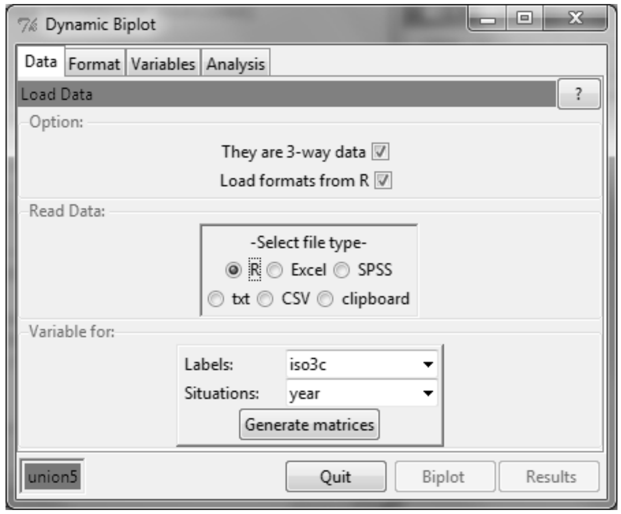

Fig. 2. Window of dyn Biplot with the loaded data

Every year, since 1995, The Heritage Foundation in association with The Wall Street Journal releases the *Economic Freedom Index* [34], which follows the theory of Adam Smith (1776). The Heritage Foundation considers that if the score of the *Index of Economic Freedom* of a country is higher than 80, it has 'truly free' economic policies; if the score is between 70-80, those policies are 'mostly free', if between 60-70, they are 'moderately free', if between 50-60, ‘mostly unfree’ and if the score is less than 50, it is considered a country whose economy is 'repressed'. In the EU there are neither ‘truly free’ countries nor economically 'repressed' ones. Greece is the only EU country that reaches the level of 'mostly unfree’ economic policies.

5.2 Materials and Methods

The data were used for this study has been obtained from ‘The Heritage Foundation’ website (www.heritage.org). It is an array of data which includes the *Index of Economic Freedom* and the ten specific indices that measure each of its components for the countries of the European continent, from which are selected the 28 countries comprising the EU. The result is a matrix that can be converted into a data cube with 28 countries (rows), and 11 variables (columns) measured in 9 periods, corresponding to the years between 2005 and 2013, both included. It was added a variable to distinguish the countries belonging to the euro area and another to qualify the score on the overall index. All scores of the freedom index range between 0 and 100.

The names used for the variables are listed below. The overall index is *Index of Economic Freedom*; in the area *Rule of law*: *Property Rights*, *Freedom from Corruption*; in the area *Limited Government*: *Fiscal Freedom*, *Government Spending*; in the area *Regulatory Efficiency*: *Business Freedom*, *Labor Freedom*, *Monetary Freedom*; in the area *Open Markets*: *Trade Freedom*, *Investment Freedom*, *Financial Freedom*.

Subsequently, using the technique of Dynamic Biplot presented above, it has been analyzed the relationship of each of the EU countries with the index of economic freedom in 2013 and the changes that took place, indices and countries, to reach that situation, obtaining their trajectories. It has been used in 2013 to be the closest of the data processed. The method allows to set the baseline for any of the periods.

5.3 Dynamic Biplot

<u>5.3.1 Step 1: static analysis</u>

It has been made the first step of the Dynamic Biplot with the data cube containing the indices of economic freedom (variables), the member countries of the EU (individuals) and index scores between 2005 and 2013, both included (situations). In this study, it is taken as reference situation the one corresponding to year 2013. Obviously, this reference matrix can be changed.

This first step corresponds to the static analysis of the data and the results are shown in Fig. 3, where it presents the HJ-Biplot analysis of the centered data. The plane 1-2 accounts for 78% of the variability. Those elements, rows and columns, which do not accumulate sufficient level inertia (<300) have been hidden. Different marks have been used for countries belonging (ϒ) or not (•) to the eurozone. Variables that are not well represented in this plane and that have been hidden are the *Monetary Freedom* and *Trade Freedom* indices. The countries that have been hidden are Cyprus, Malta and Spain.

Plane axis 1 (62.3%) is highly correlated with variables of the *Rule of law* (Property, Corruption) area. Axis 2 (16.12%) is correlated with *Labor Freedom* and, to a lesser extent, by the *Index of Economic Freedom*. Variables of the *Limited Government* (Fiscal, Gov. Spending) area, *Financial Freedom* and *Investment Freedom* are plane variables. The *Index of Economic Freedom* is correlated with variables of the *Open Markets* (Trade, Investment, Financial) and *Rule of law* (Property, Corruption) areas and with the *Labor Freedom* index, being independent of the *Business Freedom* and *Limited Government* (Fiscal, Gov. Spending) area variables.

Regarding the position of different countries according to variables they can be observed different groups; a group of countries are well positioned in relation to the *Index of Economic Freedom* and the *Open Markets* (Investment, Financial) area variables, like Ireland (IRL), Estonia (EST), UK (GBR) and Austria (AUT). Most countries with a good score on the overall index are also well positioned in the *Rule of law* area (Property, Corruption), highlighting Denmark (DNK); however, some countries with good overall score as Czech Republic (CZE) and Lithuania (LTU), have a poor score in this area, like most countries outside the eurozone, which are well positioned in the *Limited Government* area. Another group of countries stand out regarding *Business Freedom*, like Denmark (DNK), Sweden (SWE), Finland (FIN), Netherlands (NLD), Belgium (BEL) and Germany (DEU). Countries like Croatia (HRV), Italy (ITA), Portugal (PRT), Slovenia (SVN) and Greece (GRC) get bad scores in almost all areas and especially in *Labor Freedom*. France (FRA) meets poorly in the *Limited Government* area (Fiscal, Gov. Spending) and *Labor Freedom* index getting its best score in *Business Freedom*.

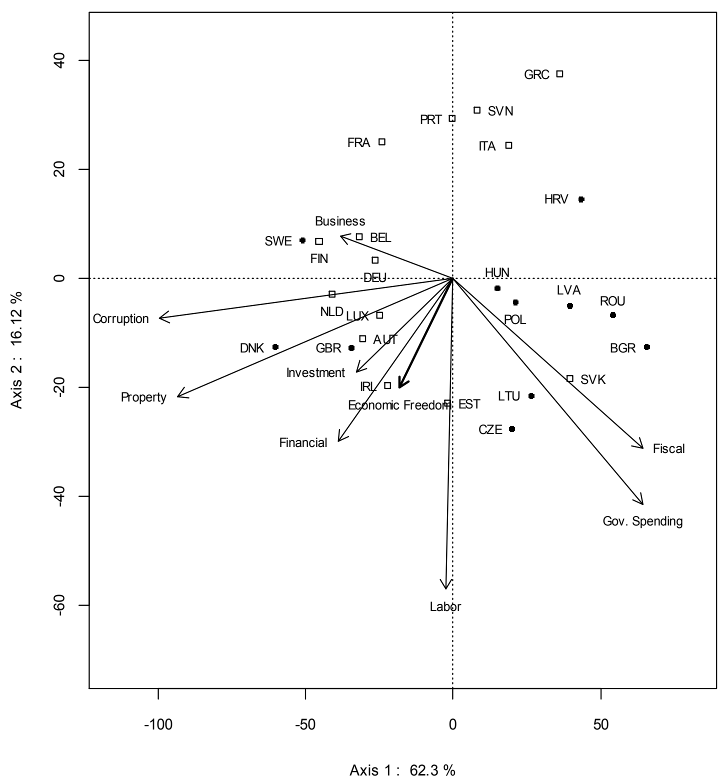

Fig. 3. Results of Step 1 - Static HJ-Biplot analysis of economic freedom indices, year 2013. It distinguishes between countries in the euro area (ϒ) and non-euro area (•)

5.3.2 Step 2: dynamic analysis

The second step of the Dynamic Biplot analysis allows us to study the dynamics of the model by observing the trajectories of both, economic freedom (Fig. 4) and countries (Fig. 6). Each trajectory is the path followed by every element up to the position corresponding to the reference situation, in 2013.

Zooming in Fig. 4 allows us to have a better look at the trajectories (Fig. 5). Similarly one could perform zoom rest of the figure.

In Fig. 5 there is a short path for the Index of Economic Freedom with a decrease up to year 2007 followed by a slight recovery. There is also a departure from Rule of law (Property, Corruption) area variables. Freedom from Corruption and Property Rights (Property) variables show a decline up to 2010 and a subsequent recovery to the current situation; it is also seen a shift towards the global index variable.

The trajectory of the *Financial Freedom* index which, up to year 2009, has decreased and shifted towards axis 1 followed by a decline in freedom until 2012. The *Labor Freedom* variable grew throughout the studied period. The *Investment Freedom* variable shows a decrease in almost all the studied time periods. The *Business Freedom* variable appears displaced towards the independence of the *Index of Economic Freedom*.

The variables of the *Limited Government* area, *Government Spending* freedom grew until 2007, declining after up to 2010 and recovering ever since; *Fiscal Freedom* has had an erratic trajectory with a trend to growth, although in the last period has decreased slightly.

Country trajectories are represented in Fig. 6. The label for every country is located on the reference situation, in this case, at the end point.

Results show that Sweden (SWE) has increased its *Business Freedom* until 2009 declining thereafter but increasing the *Index of Economic Freedom*. Finland (FIN) and Belgium (BEL) have trajectories with ups and downs but in opposite directions. Germany (DEU) has also improved its global index, like France (FRA), although at different levels. Portugal (PRT) suffers decline. The trajectories of Italy (ITA) and Greece (GRC) show a strong decrease in economic freedom, while the newly incorporated Croatia (HRV), Poland (POL) and Romania (ROU) have great increments. Slovenia (SVN) has ups and downs. The Czech Republic (CZE) shows the long trajectory, increasing the *Index of Economic Freedom*, *Labor Freedom*, *Fiscal Freedom* and *Government Spending* scores. Trajectories of other countries have ups and downs.

Zooming in Fig. 6 allows us to have a better look at the trajectories (Fig. 7). Similarly one could perform zoom rest of the figure.

Fig. 7 corresponds to countries with more economic freedom. While Denmark (DNK) and the Netherlands (NDL) are stable, Austria (AUT) has a shift to *Investment Freedom*, *Financial Freedom* and the *Index of Economic Freedom*. United Kingdom (GBR) and Ireland (IRL) have major setbacks in the *Index of Economic Freedom* and in *Labor Freedom*.

6. DISCUSSION

In this paper it has been analyzed the relationships between the indices that measure the various expressions of economic freedom and the positioning of the EU member countries regarding to them. Although most EU countries get good scores in the *Index of Economic Freedom,* not all the areas are equally developed. The *Rule of law* area (zone non-euro), especially the indicator *Government Spending* (both areas), and the *Labor Freedom* (eurozone) are those with lower scores.

It has been seen in our analysis that the EU has well developed *Property Rights*, being this index one of the indicators that helps to discriminate among different countries. However, concerning intellectual property policies, a law reform is needed in order to adapt to the reality we are living in, being a unique opportunity for the economic development of the EU [35].

The anticorruption policy, which is another indicator that contributes to discrimination among countries, also gives the desired results [36]. Clark Williams and Seguí-Mas [37] say that corruption is one problem that must be cut down in all societies. The EU, European Council [38], aims to promote policies against corruption by adopting the ‘Stockholm Programme’ although not all EU countries get the same results against corruption as showed in this study and in those carried out by other authors [39–41].

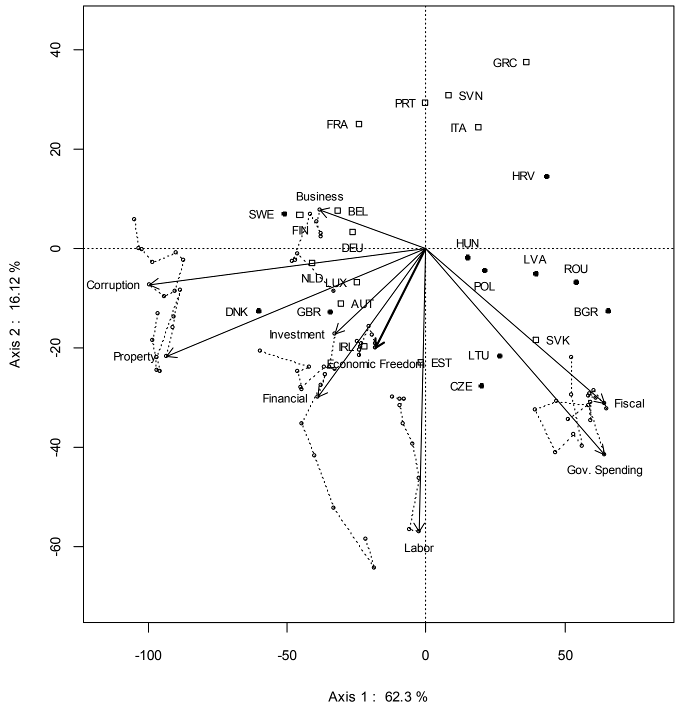

Fig. 4. Step 2 dynamic biplot: Trajectory of indices of economic freedom, ending in the reference year, 2013

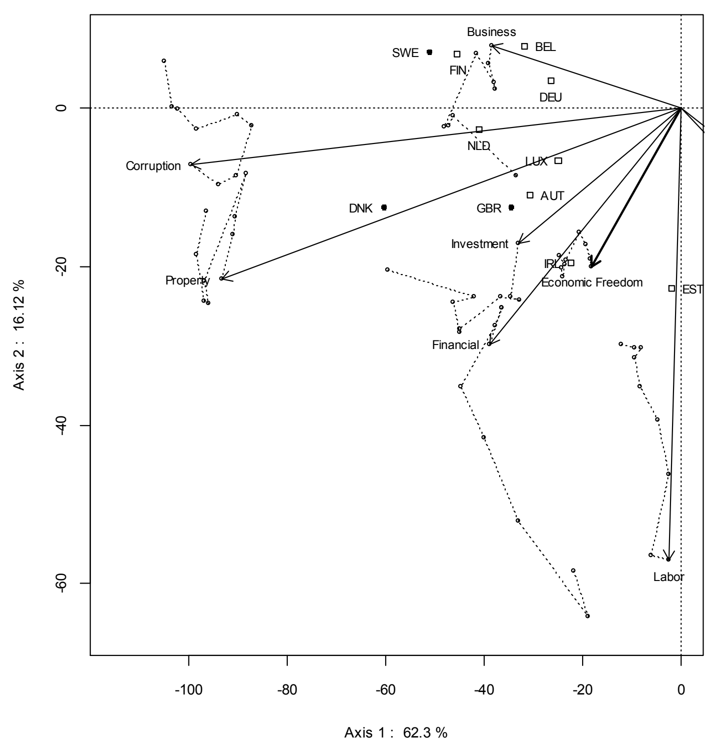

Fig. 5. Zoom to better see the trajectories of the variables, each path ends in the reference year 2013

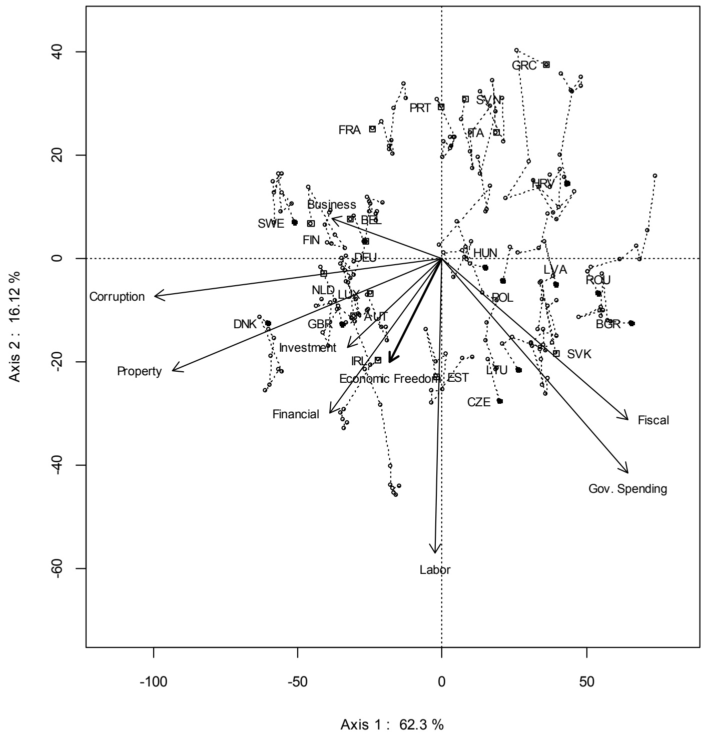

Fig. 6. Step 2 dynamic biplot: Trajectory of countries, ending in the reference year, 2013

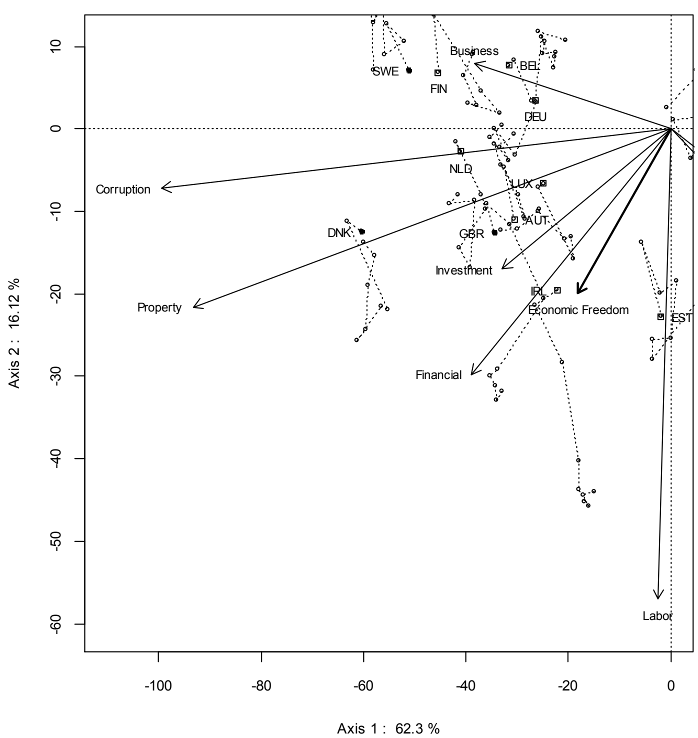

Fig. 7. Zoom to better see the trajectories of the countries, each path ends in the reference year 2013

The EU hosts some of the most advanced economies, whereas, even with common EU policies, some others are not very developed. But as you can see in Europa–Press Releases [42], the Commission is imposing to most countries an Excessive Deficit Procedure (EDP). For the EU, our analysis shows that economic freedom is barely influenced by measures to restrict governments. We should not forget that all countries except Greece, have a good score on the *Index of Economic Freedom*.

As indicated by Miller et al. [34] in *the 2013 Index of Economic Freedom*, France, Greece, Italy, Portugal and the UK had similar scores two decades ago, while Sweden, Germany, Czech Republic, Poland, Romania and Bulgaria had their best values in 2013. In our analysis we have seen that, for the analyzed period, France and Portugal improve their score, while Greece, Italy and Britain worsen it.

The *Index of Economic Freedom* of The Heritage Foundation has been used to apply various techniques. Tausch [43] used it along with other variables with ordinary least square standard regression analysis to analyze the four freedoms: goods, capital, labor and services of EU. Yerelí [44] discussed the economic freedom of countries aspiring to EU membership, concluding that "most of the transition economies which joined the EU after 2004, have a regular improvement on their level of economic freedom" an issue that we have seen with the case of Croatia.

In addition to the *Index of Economic Freedom* developed by The Heritage Foundation, there are other similar indices, as the one developed by the Fraser Institute of Canada or The European Economic Freedom Index [45]. Using these indices, authors as Sell [46] make a classification of EU countries after the 2004 expansion. Hall et al. [47] discussed the differences in economic freedom, concluding that "we find evidence that the European Union was, on net, positive for economic freedom… Further research is needed to understand exactly how EU membership affects economic freedom". Our study poses an advance on this line of research.

7. CONCLUSIONS

The Dynamic Biplot developed in this paper and applied to the data presented allows us to reach the following conclusions:

The Dynamic Biplot method has proven to be a new approach to data processing 3-way. Its use in conjunction with the developed dynBiplotGUI package, obtains results easy to represent and interpret.

The *Index of Economic Freedom* is correlated with the *Regulatory Efficiency* and *Rule of law* areas and *Labor Freedom* index, being independent of the *Business Freedom* and *Limited Government* area variables. The variables of the *Rule of law area* (*Property Rights* and *Freedom from Corruption*) are the ones that better discriminate among EU countries. Both variables have a shift toward the *Index of Economic Freedom*.

Countries outside the eurozone enjoy higher economic freedom although some of them have weaker economies than others in the eurozone, such as France and Italy. Most countries with good score on the overall index, have also a good performance in the *Rule of law* area and most countries in the non-euro zone are well positioned in the area of *Limited Government.*

Countries like Italy, Portugal, Slovenia and Greece have bad scores in most areas, especially in *Labor Freedom*. France and Croatia also have little *Labor Freedom* but perform better in *Business Freedom* and the *Limited Government* area respectively.

Croatia, recently incorporated to the EU, shows an improvement in its economic freedom trajectory.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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